DEFLECTIONS OF REINFORCED CONCRETE BEAMS USING VARIABLE MOMENT OF INERTIA

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Dedication

To My Father

My Mother

My Fiancée Moayad

With hope that this project is well done

Wafa

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A lot of work, effort and dedication have been granted to realize this thesis. This achievement wouldn't have been carried out without continuous supervision, instruction, offering inspiration and advice.

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Thanks to all my friends for standing up for me.

Thank you All.

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Notation

A_s	area of tension steel reinforcement
A'_s	area of compression steel reinforcement
b	width of compression face of member
b_w	width of web in flanged beam
C_t	creep coefficent at any time t
d	effective depth of section, or distance from extreme compression fiber to centroid of tension steel
E_c	modulus of elasticity of concrete beam
f'_c	compression strength of concrete
f_y	yield strength of steel
I	moment of inertia (second moment of area) of a section
Icr	moment of inertia of cracked transformed section
I_{e1}, I_{e2}, I_m	moment of inertia at two ends of span and at midspan
I_e	effective moment of inertia
I_g	moment of inertia of gross concrete section, neglecting steel
M	bending moment
M_a	maximum service load moment (unfactored moment) at the stage for which deflections are being considered
M_{cr}	cracking moment

M_{e1}, M_{e2}	beam end moments
ρ	tension steel ratio
ρ'	compression steel ratio
W	total load on a span
w	uniformly distributed load
β_1	ratio of depth of stress block a to the distance between neutral axis and extreme compression fiber c
ε_0	unit strain
φ	creep coefficent
Φ	mean curvature

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Abstract

This thesis proposes a procedure for predicting immediate deflections of reinforced concrete beams subjected to uniformly distributed loads by using variable moment of inertia along the beam length. A comparison is made between the results obtained by thesis approach and those determined by ACI 318M-99.

Various parameters affecting immediate deflections of beams are considered. Those include effects of span, L; tension steel ratio, ρ; compression steel ratio, ρ′; applied load, w;concrete compressive strength, f_c'; steel yield strength, f_y; beam types and beam cross-sections.

The adequacy of the proposed approach is checked by comparing calculated immediate deflections with those calculated according to the ACI. It is shown that the ACI provisions are more conservative almost in all cases than the proposed procedure, except in continuous beams with high ratios of tension steel.

The difference between the results of the two approaches is affected by several parameters. The study demonstrates that both approaches give very close results in the cases of simply supported beam, cantilever beam 2-span continuous

beam and 3-span continuous beam with tension steel ratio $\rho < 0.75\ \rho_{max}$. In other cases the results are still close.

1.Introduction

1.1 General

Structural designs are based on economy, strength, serviceability and durability. Economy should be the primary objective of a design, whereas strength and serviceability must be ensured in a design. Since we do not have adequate technical information on durability, it is often satisfied in an empirical manner.

It has been the belief of engineers of the past generation that the above design requirements are best satisfied by controlling working stresses. Concrete with a compressive strength f_c' of 11 to 21 MPa (1.5 to 3.0 ksi) and reinforcement with a yield stress of 230 to 280 MPa (33 to 40 ksi) were predominant in the earlier decades of the past century. The use of these materials with conservative allowable stresses, along with the working stress method resulted in large stiff sections having small deflections (Purushothaman, 1984).

The widespread use of the strength design method in recent years, taking into consideration the nonlinear relationship between stress and strain in concrete, has resulted in smaller sections than those designed by the working stress method. The use of steel up to a yield strength of 560 MPa (80 ksi) and the use of high strength concrete result in smaller sections and a reduction in the stiffness of the flexural member and consequently increases its deflection (Hassoun, 1998). Therefore, deflections and deflection cracking have become more severe problems than they were a few decades ago (McCormac, 1986). It is important to recall that loads imposed on structures produce forces in individual members and hence stresses. These stresses, in turn, result in strains, deformations and deflections, the behavior of a truss is a typical example of this process and it is clearly evident that deflections are the end products of a loading process (Purushothaman, 1984).

The permissible deflection is governed by the serviceability requirements for the structure, such as the amount of deformation that can be tolerated by the interacting components of the structure. Excessive deflection of the member may not in itself be detrimental, but the effect on structural components that are supported by the deflecting member frequently determines the acceptable amount of deflections (Wang and Salmon, 1998).

1.2 The Deflection Problem

Proper design of reinforced concrete beams requires that they should have adequate stiffness as well as strength. Under service loads, deflections must be limited so that attached nonstructural elements, (e.g. partitions, pipes, plaster ceilings and windows) will not be damaged or rendered inoperative by large deflections.

Design for deflection has not kept pace with design for strength, since deflection computations are internally difficult and time-consuming due to the following (Purushothaman, 1984):

- The influence of creep, shrinkage, temperature and also cracking.
- Deflection computations must be reasonably accurate, since overestimates and conservatism can lead to large size structural members.
- In the past building materials such as lime and mild steel were more pliable and the allowable stresses were lower. With the advent of high strength steel and concrete, allowable stresses have increased, shrinkage and creep effects have become important, and hence deflection check has become necessary even when structural components are designed by the working stresses method.
- Compression steel reduces creep and shrinkage deflections up to 20 to 30% of the short term deflection.
- Excessive deflections indicate a tendency towards undesirable vibrations.
- The age of concrete at time of loading has an important effect on deflections.
- Ambient weather and initial curing have significant influence on subsequent deflections.
- Incremental and total deflection limits should both be set for control of deflections.
- Distress of non-load bearing, nonstructural elements attached to flexural elements should be considered.
- Large deflections can result in failure due to instability, even when the stresses are within the specified limits.

- The assumption of a fully cracked section is usually conservative. The transformed area of reinforcing steel in uncracked sections may not always be ignored as it can increase the moment of inertia.
- The modulus of elasticity and modulus of rupture should be realistically estimated and used.

1.3 Deflections and Design Values

Excessive deflections and deformations can impair the appearance and efficiency of a structure and cause discomfort or alarm to the occupants. The maximum deflections which are permitted by the ACI Code under normal working loads are given, usually in terms of span or height. Experience has indicated that deflections are likely to be satisfactory if certain limiting span to effective depth ratios are not exceeded (Syal and Goel, 1984).

Limitations on deflection are somewhat arbitrary, historically L/360 has been the accepted limit to prevent the cracking of plaster ceilings. Other limits should be considered as guidelines, with the designer having the responsibility for evaluating the possible adverse effect of excessive deflection in any given situation.

A report by ACI Committee 435 (Park and Pauly, 1975) on allowable deflections classifies effects of deflections under four broad headings, as follows:

1.3.1 Sensory acceptability

Sensory acceptability tends to be a matter for personal judgment and depends a great deal on the social background of the users and the type of structure. Under this heading come visual effects such as sagging beams or droping cantilevers, tactile effects such as vibration due to dynamic effects of live load and wind, and auditory effects such as noise from vibrations. Deflection limits on sensory acceptability are difficult to establish because of the variability of personal opinion.

1.3.2 Serviceability of the structure

Serviceability limits are related to the intended use of the structure. Examples in this category are roof surfaces that should drain water, floors that should remain plane (e.g., gymnasia), and members supporting sensitive equipment. Deflection limits on serviceability are easier to define.

1.3.3 Effect on nonstructural elements

Deflections must be limited to prevent cracking, crushing, or other types of damage to nonstructural elements such as walls, partitions, and ceilings. Deflections should not prevent moving elements such as doors and windows from operating properly. Thermal and shrinkage effects may be important, as well as deflections due to gravity and lateral loads. The deflection limits to be applied depend on the type of nonstructural element and the method of installation.

1.3.4 Effect on structural elements

Deflections may need to be limited to prevent the structural behavior from being different from that assumed in the design. Examples in this category are deflections causing instability such as arches and shells or long columns, deflections causing a change in the stress system such as a change in the bearing area due to beam end rotation, and deflections causing dynamic effects that increase stresses such as resonant vibrations due to moving loads. When possible, the effects of deflections on the structural behavior should be included in the design of the element.

1.4 Approaches for Controlling Deflections

1. The use of compression steel and limiting the tension steel percentages in reinforced members. This is a method of using relatively small tension steel percentages in the design of reinforced concrete members (which in turn results in relatively deep beams to minimize deflections. The tension steel ratio $0.18f'_c/f_y$ was shown to be less than half the balanced ratio (ultimate strength design), and was judged to be sufficiently low to minimize deflection problems in most cases.

Alternatively, structural members will normally be of sufficient size so that deflections will be within acceptable limits when the tension steel reinforcement used in the positive moment zone does not exceed the following percentages of that in the balanced condition: for member not supporting or not attached to nonstructural elements likely to be damaged by large deflections--

35 percent for rectangular and 40 percent for Tor box beams, of the balanced ratio; and for members supporting or attached to nonstructural elements likely to be damaged by large deflections–25 percent for rectangular and 30 percent for Tor box beams of the balanced ratio.

The use of compression steel is very useful in reducing time-dependent deflections (Branson, 1977).

2. It is possible to design a structural element that satisfies the deflection criteria by limiting the span/depth ratio (Purushothaman, 1984). The use of maximum span-depth ratios and minimum depth is essentially an important approach based largely on experience, even when analytical methods are used to derive such limiting values. The calculations are based on selected allowable deflections on analytical procedures for determining span-depth ratios (Branson, 1977). In general, it is the collapse limit which governs the size of the member, and only in rare cases, such as very long spans, do the deflection criteria control the structural proportions.

With regard to deflection control, flexural members may be classified into two groups:

(i) Those supporting nonstructural elements which are likely to be damaged.

(ii) Those which do not have nonstructural elements which are likely to be damaged.

In the first case the total deflection and incremental deflections after the erection of partitions, etc. should be checked. In the latter case, the total deflection alone needs to be checked (Purushothaman, 1984).

The minimum thicknesses of beams and one-way slabs shall be in accordance with Table (1.1).

Table 1.1. Minimum thickness of beams or one-way slabs unless deflections are computed *

		Minimum Thickness, h		
	Simply supported	One end continuous	Both ends continuous	Cantilever
Member	Members not supporting or attached to partitions or other construction likely to be damaged by large deflections.			
Solid one-way slabs	L/20	L/24	L/28	L/10
Beams or ribbed one-way slabs	L/16	L/18.5	L/21	L/8

* ACI 318M-99 Code, Table 9.5 (a).

This method of controlling deflections is simpler than the other method in which calculated and allowable deflections are compared. This approach usually must be quite conservative and/or with limited applicability. This is increasingly true as deflection becomes more critical (Branson, 1977).

3. The use of calculated and allowable deflections. The proper control of deformations in structures involves a consideration of various displacements, deflections, rotations, and both amplitude and frequency of vibrations, etc. compared with usable limits based on integrity, serviceability, esthetic, and physiological requirements. However, the primary approach used by most engineers refers to placing limits on computed deflections.

The following allowable deflections Table (1.2) apply to reinforced concrete building members when deflections are computed by the 1999 ACI Code method.

Table 1.2. Maximum permissible computed deflections *

Type of Member	Deflection to be Considered	Deflection Limitation
Flat roofs not supporting or attached to non structural element likely to be damaged by large deflections.	Immediate deflection due to live load L	L/180
Floors not supporting or attached to non structural elements likely to be damaged by large deflections.	Immediate deflection due to live load L	L/360
Roof or floor construction supporting or attached to non structural elements likely to be damaged by large deflections.	The part of the total deflection occurring after attachment of non structural elements (sum of the long-term deflection due to all sustained loads and the Immediate deflection due to any additional live load).	L/480
Roof or floor construction supporting or attached to non structural elements not likely to be damaged by large deflections.		L/240

*ACI 318M-99, Table 9.5 (b).

4. By appropriate construction practices. In addition to the use of concrete with maximum strength and stiffness properties and minimum creep and shrinkage properties, deflections can be minimized in other ways by appropriate construction practices. One example of this is to delay form removal (to minimize creep deformation) as long as possible, and then to install partitions as late as possible, since this will tend to minimize the creep and shrinkage deflection that could cause damage to the partitions (Branson, 1977).

2.Literature Review

2.1 Introduction

With the present day use of higher strength concrete and reinforcing steel, the strength or load-factor method of design result in shallower sections (Hassoun, 1998). The problem of predicting and controlling defections of reinforced concrete flexural members, under service loads, has thus become increasingly important since the 1950s (Ferguson, 1973). As such serious comprehensive studies of the deflection problem in reinforced concrete structure began about fifty years ago.

2.2 Deflection of Reinforced Concrete Beam

Many researchers have investigated the trends that control the deflection problem in reinforced concrete structures. Some of them considered the deflection prediction for concrete beams, others discussed the causes of wrong estimation of deflection, while other researchers considered long-term deflection of reinforced concrete beams under constant loads.

2.2.1 Deflection prediction for reinforced concrete beam

Prediction of immediate and long-term deflections is important in the design of a concrete member for satisfactory performance during its use. Usually, structural engineers and structural design codes pay more attention to safety against failure than to quality of structures under service conditions.

However, unsatisfactory performance such as excessive deflection or cracking occurs more frequently than structural collapse (Ghali and Azarnejad, 1999). Therefore, many studies have been made to predict immediate and long-term deflections.

Sherif and Dilger (1998) critically reviewed the provisions of several codes for the deflection calculations of normal and high strength reinforced concrete beams. Both short and long-term deflections are discussed. Tests are used to assess the calculation methods suggested by the codes. These methods are the effective moment of inertia approach, the mean curvature approach which determined the deflection of a member by integrating the curvature Φ at a number of sections and the bilinear method which based on the observation that, for the serviceability limit state, the moment-deflection relationship may be approximated by a bilinear relation. A parametric study is carried out to investigate the effect of the level of loading, shape of bending moment, and reinforcement ratio on the predicted deflection.

The following are the most important conclusions for instantaneous and long-term deflections:

1. The main shortcoming of the effective moment of inertia approach is that it does not account for the shape of the moment diagram along the member in determining the effective moment of inertia. The mean curvature approach does this indirectly by calculating the deflections by integrating the mean curvature at several sections along the beam.

2. For beams with a low reinforcement ratio and an applied moment close to the cracking moment, both the effective moment of inertia approach and the bilinear method underestimate the deflections considerably.

3. Although the bilinear method includes the least computational efforts when compared with the effective moment of inertia or the mean curvature method, the accuracy of the predicted deflections is not substantially affected, thus, making the bilinear method an attractive one for quickly estimation of deflections.

4. The ACI 318M-99 approach for calculating the long-term deflections overestimates the ratio of long-term to initial deflections, especially for high strength concrete beams. Sherif and Dilger proposed to apply a correction factor to the long-term deflection multiplier of the ACI 318M-99 Code which accounts for the effect of concrete strength on long-term deflection.

5. The mean curvature approach and the bilinear method result in long-term to initial deflection ratios that agree very well with test results.

6. For beams without compression reinforcement an increase in the concrete strength results in a decrease in the ratio of the long-term to initial deflections. However, for beams with compression reinforcement, the ratio of long-term to initial deflections is independent of concrete strength.

Ghali and Azernejad (1999) developed a rational analysis model which satisfies the requirements of equilibrium and compatibility that reduced the error in prediction of immediate and long-term deflections of reinforced concrete members. They compared this model with experimental values reported by Christiansen (1988), Corley and Sozen (1966), and Bakoss (1982) et al. The study showed that deflection of a member can be determined more accurately from the values of the mean curvature at a number of sections using simple geometrical deflection-curvature relationships and that long-term deflection cannot be predicted accurately by the use of the multiplier λ used by the ACI 318M-99 code because it does not include several parameters that influence deflection. The study indicates the order of magnitude of the change of deflection with change of concrete strength.

2.2.2 Deflection calculation for reinforced concrete structures. Why Do We Sometimes Get It Wrong?

The simplified procedures contained in ACI 318M-99 for calculating the deflection of beams and slabs are inadequate in some situations. The calculated deflection is often significantly less than the actual deflection, and serviceability problems resulting from excessive deflection are not uncommon for structures designed in accordance with the code (Gilbert, 1999).

Gilbert (1999) presented and evaluated three alternative methods for improving the calculation procedure adopted by ACI 318M-99. Alternative 1,accounts for the breakdown of tension stiffening under long-term or cyclic loads, while, alternative 2 includes the shrinkage induced tension in the estimation of the cracking moment. Alternative 3, however, accounts for the actual creep and shrinkage and characteristics of concrete, by calculating the creep deflection and shrinkage deflection separately, and by so doing, can be used to obtain reliable estimates of the final deflections.

Stewart (1996) developed a probabilistic model to estimate immediate, creep, shrinkage deflections and the probabilities of serviceability failure of reinforced concrete beams sized according to the span-to-depth ratio serviceability requirements of ACI Code. The results suggest that probabilities of serviceability failure are not consistent across a range of beam spans and that the span-to-depth ratio serviceability requirement specified in the ACI Code produce significantly different risks of serviceability failure.

Ghali (1993) indicated that calculating the immediate and long-term deflections of reinforced concrete members can be inaccurate for two main reasons. The first is the uncertainty of the material parameters: elasticity modulus, creep coefficient, shrinkage and tensile stress of concrete. The second is the use of an inadequate method of analysis. The study showed that the approach of the code yields accurate prediction of the immediate deflection in some cases, but this is not the case in other practical applications, for e.g., when the reinforcement ratio is low, when the maximum moment is not substantially greater than the cracking moment and when the bending moment is constant over the major part of the member. No alternative equation is suggested for I_e because such an equation is dispensable. The following changes to the ACI 318M-99 Code are suggested:

1. The equation for the effective moment of inertia is to be replaced by an equation for the mean curvature Φ_m.

2. The equation for multiplier λ is to be omitted. Instead, the requirements of compatibility and equilibrium in the analysis are to be stated in the code.

2.2.3 Deflections of reinforced concrete beams under constant loads

Deflection analysis of concrete structures is relatively complex subject of structural engineering. The main issue herein is the concrete material behavior, with which researchers associate parameters such as concrete mix proportions, humidity, temperature, size and load duration. Further technical difficulties arise due to the spatial and temporal variations of concrete properties and the differences in concrete deformation behavior under tension and compression (Alwis, 1997).

Alwis (1997) proposed a method of constructing a time-dependent bilinear moment-curvature curve for reinforced concrete beams and demonstrated its use for estimating long-term deflection of statically determinate beams under constant loading. The moment-curvature relationship defined herein is meant for a beam length that is subjected to a constant moment profile as opposed to a section subjected to a varying moment. The time-dependent concrete material behavior is assumed to be characterized by the shrinkage strain and an effective modulus. The beam behavior is then derived by considering the uncracked and fully cracked section. A linearized moment-curvature relationship is adopted for the cracked beam segments in order to represent the tension stiffening effect. The proposed moment-curvature description formally links the load-dependent deformation to shrinkage which is fundamentally a stress-independent deformation measure. Similarly, the stress-dependent deformation of the concrete elements due to instantaneous and creep effects is formally linked to the beam deformation under zero loading. This is a departure from the current approaches of calculating separate deflection terms based on shrinkage and effective stiffness, where the fundamental stress-independent and stress-dependent deformation measures are considered separately when calculating deflections.

Nie and Cai (2000) developed an analytical model that incorporates time-dependent effects (creep and shrinkage) to predict the long-term deflection of cracked reinforced concrete beams under sustained loading. The deflection model included both bending and shear effects. Experimental studies were conducted to verify the analytical model. It showed that the time-dependent deflection increment under sustained loading for a duration of three months varied from 48 to 88% of the initial deflection and that temperature and relative humidity might have significant effects on the time-dependent deflection increment. The study established a method of calculating the creep coefficient φ based on the strain measurements. This provides an approach to predicting and calibrating the values of φ specified in code specifications. The simplified calculation of ACI 318M-99 predicts larger time-dependent deflection than test measurements. It is noted that the analytical results were verified with the specimens that have a reinforcement ratio of 1.42 or higher. Nie and Cai concluded that for members with low reinforcement ratio, the contribution of concrete stiffening may be significant and ignoring the tensile strength of concrete may be inappropriate.

3.Background and Theoretical Survey

3.1 Introduction

It should be emphasized that the minimum thicknesses shown in Table (1.1) proposed by the ACI 318M-99 apply only to members not supporting or attached to partitions and other constructions likely to be damaged by deflection. When a large deflection is likely to cause such damage, it must be computed whether or not the minimum thickness requirement is satisfied.

The practicing engineer can expect deviations greater than 30 percent between predicted and measured deflections of beams constructed under actual field conditions. A study of deflections of reinforced concrete beams must account for the instantaneous elastic deflections as loads are first applied, as well as for the long-term deflections that develop due to creep and shrinkage and continue to increase over a period of several years. Under a constant value of load, by the time long-term deflections reach their maximum value, they are generally of the order of twice the magnitude of the initial elastic deflections (Leet, 1997).

3.2 Short-Term Deflection

Elastic theory equations for deflections assume linear behavior between stress and strain are used in calculating instantaneous deflections caused by undergoes some increase in strain and stress because of the decreased moment lever arm with time.

2. **Shrinkage of Concrete.** Concrete shrinkage causes curvatures and deflections in the same direction as those caused by gravity loading. Shrinkage and creep deflections are complementary, their combined value estimated in approximate calculations with a single time-dependent factor applied to the initial deflection. Such a procedure is used in the ACI 318M-99 Code.

3. **Formation of new and widening of earlier cracks.** Laboratory tests showed that the formation of new cracks during sustained loading seems to depend on the development of earlier cracks during the initial loading stage. About half of the cracks occur at initial loading and the remainder during sustained loading.

4. **Relaxation of tensile stresses in concrete.** Tensile stresses in the concrete between cracks will be reduced by relaxation, resulting in an increase in curvature and deflection with time. It has been shown that the long-term curvature due to creep of concrete in tension, as a percentage of the total creep curvature, may increase from about 10 percent for high reinforcement percentages to the theoretical value of 50 percent for unreinforced (uncracked) concrete.

5. **Movement of the neutral axis.** The dominant effect of movement of the neutral axis is downward due to creep.

6. **Compression steel.** Compression steel has the effect of significantly reducing both creep and shrinkage deflections. Such reinforcement is

advocated by some engineers for no other reason than deflection control, especially in cantilever beams and slabs and other cases where deflections are frequently critical or may be critical.

7. **Effect of repeated load cycles**. The effect of repeated loading on the time-dependent response of simply and doubly reinforced beams has been studied and detailed calculations to take this effect into account normally require more information than is usually available.

8. **Moment redistribution due to cracking, creep and shrinkage in statically indeterminate structures**. This combined effect in statically indeterminate structures causes additional initial and time-dependent deflections that can readily be taken into account by numerical procedures. The combined effect will contribute to the total deflection normally by a few percent.

3.4 The ACI-Code Approach to Deflection Estimation

The first ACI code provision on deflections, other than load tests, appeared in 1963, with an expanded provision included in the 1999 ACI code. The codes take an overall approach in terms of the immediate deflection plus the expected overall percentage increase with shrinkage and time effect. The immediate deflection which is the starting point is quite sensitive to whether the member is uncracked or cracked and if cracked how severely cracked, and shall be computed with the modulus of elasticity E_c for concrete and with the effective moment of inertia I_e (ACI 318-99).

deflections must take these variations into account (McCormac, 1986). Today the ACI 318M-99 Code uses a formula developed in 1963 by Branson. This empirical expression is used for the effective moment of inertia of any particular cross section of a beam. This moment of inertia is an average value and it is a function of the bending moment, section properties and concrete strength in a form that includes the extent of cracking caused by varying moment throughout the span. The effective moment of inertia of the concrete section is given by:

$$I_e = (M_{cr} / M_a)^3 I_g + [1 - (M_{cr}/M_a)^3]\, I_{cr} \leq I_g \qquad (3.1)$$

Where:

M_{cr} = cracking moment = $f_r\, I_g/y_t$

f_r = modulus of rupture = $0.7\sqrt{f_c'}$ MPa

M_a = maximum service load moment acting at the condition under which deflection is computed.

I_g = moment of inertia of gross section (without considering the steel).

I_{cr} = moment of inertia of transformed cracked cross section.

Equation (3.1) should be used when $1 \leq M_a / M_{cr} \leq 3$. If $M_a / M_{cr} > 3$ the cracking will be extensive and I_e can be taken equal to I_{cr}. If $M_a / M_{cr} < 1$, no cracking is likely and I_e can be taken as equal to I_g (Leet, 1997).

3.4.2.1 single value of effective moment of inertia for practical use

As an approximation, a single value of effective moment of inertia is suggested for practical use when the variable I results from the variation in the extent of tension concrete cracking. Three methods have been suggested (Wang and Salmon 1979).

1. Midspan value alone:

$$I_e = I_m \qquad (3.2)$$

Where I_m is the effective moment of inertia at midspan for simply supported and continuous spans, and at the support section for cantilevers. This is the simplest method.

2. Weighted average:

In this method the adjusted I is obtained by weighing the moments of inertia in accordance with the magnitudes of the end moments. The following weighted average expression has been recommended by ACI committee 435 as giving a somewhat improved result over the use of the midspan value alone. For spans with both ends continuous:

$$\text{Average } I_e = 0.7\, I_m + 0.15\, (I_{e1} + I_{e2}) \qquad (3.3)$$

For spans with one end continuous:

$$\text{Average } I_e = 0.85\, I_m + 0.15\, I_{e1} \qquad (3.4)$$

3. Simple average:

With this assumption the I to be used in the average I is:

$$\text{Average } I_e = (0.5\, (I_{e1} + I_{e2}) + I_m)/2 \qquad (3.5)$$

Where I_{e1}, and I_{e2} are the effective moments of inertia at the two ends of the span. The use of both I_{e1}, and I_{e2} is appropriate only when there are end moments at both ends.

For uniform loading on continuous spans Eq. (3.3) representing weighted average is slightly more accurate than using the midspan value only, but for concentrated loads it is less accurate. When a simple average value is used as permitted by ACI 318M-99, it should be done in accordance with Eq. (3.5), rather than taking the sum of I_m, I_{e1}, and I_{e2} and dividing it by three. For a single heavy concentrated load, averaging reduces accuracy. In this case Eq. (3.2) representing midspan value alone should be used in such cases.

3.4.3 Short-Term deflections in design

Throughout the history of reinforced concrete construction, computation of short-term deflection has usually involved using either transformed cracked section or gross uncracked section. In either case this equation is suitable for short-term deflection calculations (Wang and Salmon, 1998).

$$\Delta = \beta_a (ML^2/E_cI_e) \qquad (3.6)$$

Where:

β_a = coefficient based on load and support conditions

I_e = effective moment of inertia

E_c = modulus of elasticity of concrete, $E_c = 4700\sqrt{f_c'}$ MPa

3.4.4 Long-Term deflections in design

Long-term or sustained loads, however, cause large increases in the deflections due to shrinkage and creep. The factors affecting deflection increases include humidity, temperature, curing conditions, compression steel content, ratio of stress to strength and the age of the concrete at the time of loading.

If concrete is loaded at an early age, its long-term deflections will be greatly increased. Excessive deflections in reinforced concrete structures can very often be traced to the early application of loads. The creep strain after about five years (after which creep is negligible) may be as high as four or five times the initial strain when loads were first applied, seven to ten days after the concrete was placed. This ratio may only be two or three for loads when the loads were first applied, three or four months after concrete placement.

Because of the several factors mentioned, the magnitudes of long-term deflections can only be estimated. The ACI 318M-99 Code states that to estimate the increase in deflection due to these causes, the part of the instantaneous deflection that is due to sustained loads may be multiplied by the empirically derived factor λ and the result added to the instantaneous deflection.

$$\lambda = \frac{\zeta}{1+50\rho'} \qquad (3.7)$$

Where:

ζ = time-dependent factor that may be determined from Table (3.1)

ρ' = ratio of compression steel = A_s'/bd

A_s' = area of compression steel

b = width of cross section

d = effective depth of the cross section

The full dead load of a structure can be classified as sustained load, but the type of occupancy will determine the percentage of live load that can be called sustained. For an apartment house or for an office building, perhaps only 20% to 25% of the service live load should be considered as being sustained, whereas perhaps 70% to 80% of the service live load of a warehouse might fall into this category (McCormac, 1986).

Table 3.1. Time factor for sustained loads *

Duration of sustained load	Time-dependent factor ζ
5 years or more	2.0
12 months	1.4
6 months	1.2
3 months	1.0

*ACI 318M-99, Table 9.5.2.5

3.4.5 Practical complications

The ACI Code procedure covers the simplest possible case, an immediate sustained load and its time effect, plus a later live load regarded as transient. The designer will recognize practical complications. How much will the immediate deflection and time effects be increased by other cracking induced by normal construction loading? Would not much of the time effect be based on cracking that goes with the normal full live load, even if such loading is itself transient? When live load is heavy, does this not imply manufacturing or storage usage, where much of this live load causes creep starting after the early initial period, which means this portion of the creep starts on an older concrete?

For the heavy live load case many complications are to be expected. Particularly it is important to note that in the long run the dead load deflection becomes that based on the maximum cracking condition plus any accumulated time effects. Hence, early smaller calculated deflections are useful only for evaluating the maximum increase in deflections which partitions must accept and evaluating time effects. Early construction loads or transient live loads will induce cracking which lowers I_e even where it is not apart of the sustained load. It appears that the code procedure might give an overly precise value of the early I_e, one which may well be too high in view of the actual physical complications (Ferguson, 1973).

3.5 More Accurate Methods for Calculating Deflections

The report of ACI Committee 435 gives a summary of methods available for calculating deflections and comparing their accuracy (Park and Paulay 1975). The ACI Code method may normally lead to sufficient accuracy for design purposes; if accuracy greater than ±20% is required, however, a more comprehensive analysis could be carried out. Such an analysis can only be justified if experimental data are available for the modulus of rupture and the modulus of elasticity of the concrete, and for the shrinkage and creep characteristics of the concrete in the environment in which the member is in service. Some suggestions of ACI committee 435 for more accurate calculations of immediate deflection, and methods due to Branson for calculating the additional long-term deflections due to creep and shrinkage, as shown below.

3.5.1 Immediate deflections

Almost all beams designed as simply supported spans have some restraint against rotation at the ends. A small moment will reduce the central deflection significantly. Therefore, some assessment could be made of the degree of end restraint available from elements such as masonry walls and concrete topping and included in the deflection calculations.

The modulus of rupture and the modulus of elasticity for the deflection calculations could be obtained from the concrete used for the structure. For example, the modulus of elasticity could be calculated from the average measured cylinder strength rather than from the specified minimum cylinder strength used in the design. The modulus of rupture may exceed the value recommended by the code for use in calculating M_{cr} and the average measured value could be used.

Also possible is more realistic assessment of the manner in which non-structural elements, particularly walls, affect structural behavior. For example, partition walls may span from end to end when the structural member deflects, beams may come to rest on walls below, and infill walls may stiffen frames considerably.

Flanges of T beams on the tension side should be included in moment of inertia calculations. Also, the transformed area of reinforcing steel in uncracked sections should not be ignored, particularly in the case of heavily reinforced members, because it can increase the moment of inertia significantly.

In continuous members a more realistic assessment of the flexural rigidity along the member could be made, rather than simple averaging of the negative and positive moment of flexural rigidities.

Shear deflections should be accounted for when thin-webbed members are used, or when a large proportion of the shear stresses is resisted by web reinforcement resulting in diagonal tension cracks under service load conditions.

3.5.2 Long-Term deflections due to concrete shrinkage

Concrete shrinkage causes a shortage of the member that is resisted by the reinforcing steel, inducing compressive stresses in the steel and mainly tensile stresses in the concrete (Park and Paulay, 1975). Shrinkage deflection is not usually calculated separately but is combined with creep deflection, according to ACI 318M-99 Code procedures. Equations for curvatures due to shrinkage for uncracked and cracked sections can be developed using elastic theory. However, such solutions are not exact because of the difficulty of dealing accurately with the effects of concrete creep. Also shrinkage deflections are normally of the order of 30% or less of the total deflections. Hence simplified approaches suffice.

3.5.3 Long-Term deflections due to concrete creep

Long-term deflections due to concrete creep are often greater than the sum of the deflections from the other effects and therefore are of primary interest. An accurate analysis including the effect of variable loading is extremely difficult because of the need for data on the creep strain-time characteristics of the concrete, and the loading history. The rate-of-creep method or the superposition method may be used if such data is available. Usually the analysis cannot be justified and a more approximate approach is chosen.

One approximate method uses the effective modulus of elasticity of the concrete for calculating the immediate plus creep deflection. The modulus is given by $E_c/(1+C_t)$, where E_c is the modulus of elasticity at the instant of loading, and C_t is the creep coefficient of the concrete. Since the creep coefficient C_t is the ratio of the creep strain to initial (elastic) strain, it is evident that in this approach the deflection due to creep is equal to the immediate deflection multiplied by the creep coefficient. However, this approach is very approximate. Concrete creep under constant bending moment results in a significant increase in the extreme fiber compression strain, an increase in the neutral axis depth, an increase in the steel compressive stress, and a decrease in the concrete compressive stress. The tensile stress in the steel increases slightly because the lever arm is reduced (Park and Paulay 1975).

4.Thesis Approach and Parametric Study

4.1 General

As was mentioned before and according to the current codes, the immediate deflection of a cracked member can be calculated using constant effective moment of inertia I_e, given by an empirical equation (Eq. 3.1). A cracked member behaves, in general, as a member of variable cross section, because the rigidity is much reduced in the cracked zone and the amount of reduction varies along the span. The value of initial strain in concrete ε_o and curvature ψ at a section depend on the value of bending moment M, as well as the cross-sectional properties. For this reason, it is impossible to find empirical equations that give constant cross-sectional properties to allow treating the member as prismatic. Such an equation will be accurate for a particular shape of bending moment diagram and will be erroneous for others. The prediction of immediate and long-term deflections of reinforced concrete members using the equation of the current ACI Code hence, is accurate in some cases, while in others, the predicted deflections can be largely in error. Examples of such cases are when the reinforcement ratio is low and variable, when the maximum moment is not substantially greater than the cracking moment and when the bending moment is constant over the major part of the member (Ghali, 1993).

4.2 Thesis Approach

The main objective of this research is to calculate immediate deflection of reinforced concrete beams by determining the moment of inertia of three zones along the beam depending on the moment diagram under service load by considering the following zone types:

- small moment, where $M_a/M_{cr} < 1$

In this case the beam section is uncracked and $I = I_g$.

- Intermediate moment, where $1 \leq M_a/M_{cr} \leq 3$

In this case the beam section is moderately cracked and $I = I_e$.

- High moment, where $M_a/M_{cr} > 3$

In this case the beam section is cracked extensively and $I = I_{cr}$.

Then the calculated immediate deflections by using variable moment of inertia along the beam with the values of deflections calculated based on ACI 318M-99 Code and with experimental results if it is available.

A tailored software was used to carry out these calculations (Appendix B).

The selection of design parameters should be wisely picked to obtain tangible trends.

4.3 Design Parameters

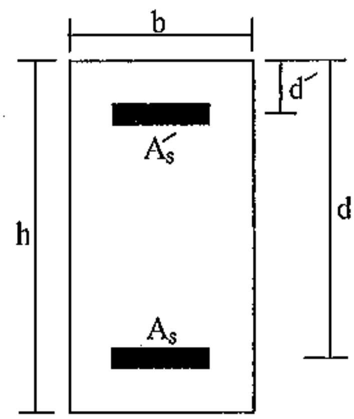
The same set of design variables are used for some studies to enable a meaningful comparison of the results, while others had different variables to assess the effect of these variables in the result.

4.3.1 Choice of beam type

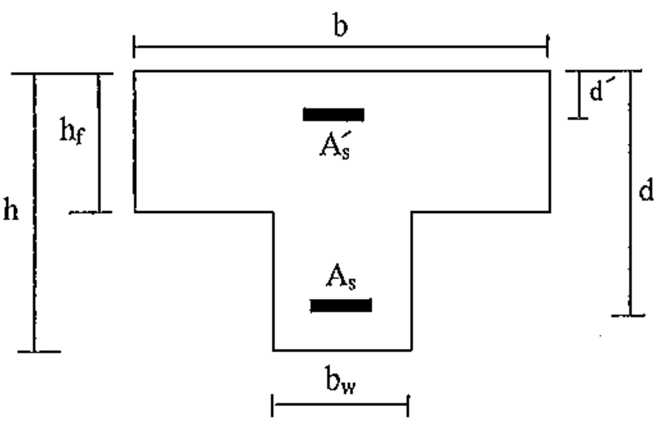
Several beam types were used in the studies: simply supported beam, continuous beam with two and three spans and cantilever beam.

4.3.2 Choice of beam cross-section

Studies consider rectangular cross-section and T cross-section, figure (4.1) shows these sections.



(a) Rectangular cross-section



(b) T cross-section

Figure(4.1) Beam cross sections (a)Rectangular cross-section (b)T cross-section

4.3.3 Types of supports

Except for cantilever beams where one end support is fixed, all supports in other beams are hinged.

4.3.4 Materials

Several values of concrete compreesive strength (21, 28, 35) MPa and steel yield strength (300, 420) MPa were used in studies to show the effect of these variables on immediate deflection values.

4.3.5 Steel reinforcement

Several ratios of tensile and compressive steel reinforcement were used, tension steel ratio ρ = (0.25, 0.5 , 0.75) ρ_{max} and compression steel ratio ρ' = (0.25, 0.5, 0.75) ρ. These ratios were used at the high moment regions.

4.3.6 Loading

Loads are assumed to be uniformly distributed along the beam length for all studies, the values of dead load (10, 15, 20)kN/m and live load (25, 35, 45)kN/m.

4.3.7 Span length

Span length were used (3, 4)m for simply supported beams, (3, 4, 5)m for 2-span continuous beams, 5m for 3-span continuous beams and 3m for cantilever beams.

4.4 Cases of Study

Four studies are performed as applications on the immediate deflection of reinforced concrete beams. These studies consider variable beam cross-section, beam type, load level, span length, concrete compressive strength, steel yield strength and ratios of tensile and compressive steel. Study number four had an experimental results to compare it with thesis and ACI values of immediate deflection.

4.4.1 Study number one

The purpose of this study is to detect the trends in immediate deflection associated to the following:

- 2-span continuous beam
- Tension steel content $\rho = (0.25, 0.5, 0.75)\ \rho_{max}$.
- Span length L = (3, 4, 5) m
- Concrete compressive strength $f_c' = 28$ MPa
- Steel yield strength $f_y = 420$ MPa
- Uniformly distributed dead load = (10, 15, 20) kN/m
- Uniformly distributed live load = (25, 35, 45) kN/m
- Six cases in this study, the first three cases for rectangular cross-section and the remaining cases for T cross-section

Tables (4.1-a to 4.2-c) show the variables used in this study.

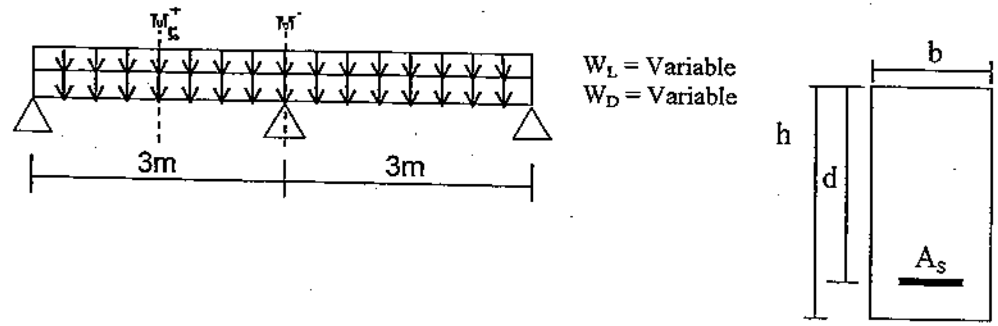
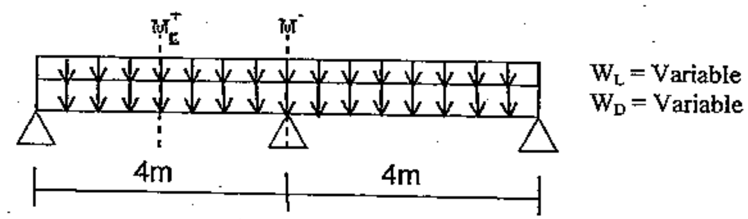


Table 4.1-a Parameters used in study number one (case 1)
2- span continuous beam (rectangular cross-section) L=3m

								M^-	M^+			
f'_c (MPa)	f_y (MPa)	ρ_{max}	ρ/ρ_{max}	ρ_t	b (mm)	d (mm)	h(mm)	A_s (mm^2)	A_s (mm^2)	w_L(kN/m)	w_D(KN/m)	w_a[1](kN/m)
28	420	0.02125	0.25	0.00531	300	335	410	534	350	10	25	38.0
			0.5	0.01063		245	320	781	365			37.3
			0.75	0.01594		200	275	956	455			37.0
28	420	0.02125	0.25	0.00531	300	400	475	638	400	15	35	53.4
			0.5	0.01063		290	365	924	440			52.6
			0.75	0.01594		240	315	1148	540			52.3
28	420	0.02125	0.25	0.00531	300	450	525	717	450	20	45	68.8
			0.5	0.01063		325	400	1036	500			67.9
			0.75	0.01594		275	350	1315	600			67.5

1 Includes own weight of beam



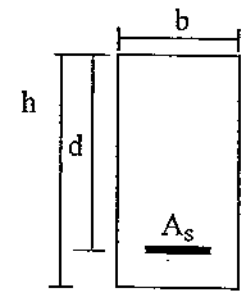
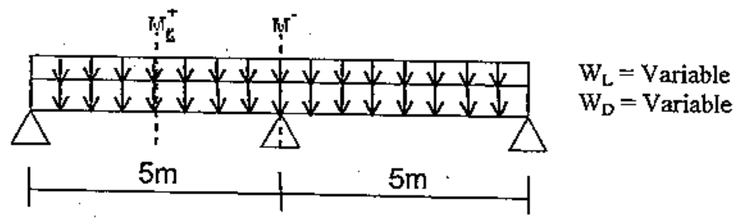


Table 4.1-b Parameters used in study number one (case 2)

2- span continuous beam (rectangular cross-section) L=4m

								M^-	M^+			
f'_c (MPa)	f_y (MPa)	ρ_{max}	ρ/ρ_{max}	ρ	b (mm)	d (mm)	h(mm)	A_s (mm^2)	A_s (mm^2)	w_L(kN/m)	w_D(KN/m)	w_a[1](kN/m)
28	420	0.02125	0.25	0.00531	300	450	525	717	355	10	25	38.8
			0.5	0.01063		325	400	1036	495			37.9
			0.75	0.01594		270	345	1291	600			37.5
28	420	0.02125	0.25	0.00531	300	535	610	853	420	15	35	54.4
			0.5	0.01063		385	460	1227	595			53.3
			0.75	0.01594		320	395	1530	725			52.8
28	420	0.02125	0.25	0.00531	300	605	680	964	480	20	45	69.9
			0.5	0.01063		435	510	1387	675			68.7
			0.75	0.01594		365	440	1745	800			68.2

1 Includes own weight of beam



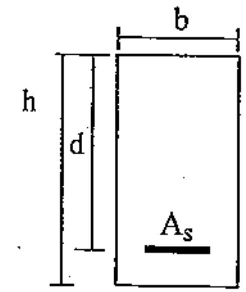


Table 4.1-c Parameters used in study number one (case 3)

2- span continuous beam (rectangular cross-section) L=5m

								M^-	M^+			
f'_c (MPa)	f_y (MPa)	ρ_{max}	ρ/ρ_{max}	ρ	b (mm)	d (mm)	h(mm)	A_s (mm^2)	A_s (mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
28	420	0.02125	0.25	0.00531	350	530	605	985	490	10	25	40.1
			0.5	0.01063		380	455	1413	675			38.8
			0.75	0.01594		315	390	1757	810			38.3
28	420	0.02125	0.25	0.00531	350	625	700	1162	580	15	35	55.9
			0.5	0.01063		450	525	1673	800			54.4
			0.75	0.01594		375	450	2092	960			53.8
28	420	0.02125	0.25	0.00531	400	665	740	1413	700	20	45	72.1
			0.5	0.01063		475	550	2019	980			70.3
			0.75	0.01594		400	475	2550	1170			69.6

1 Includes own weight of beam

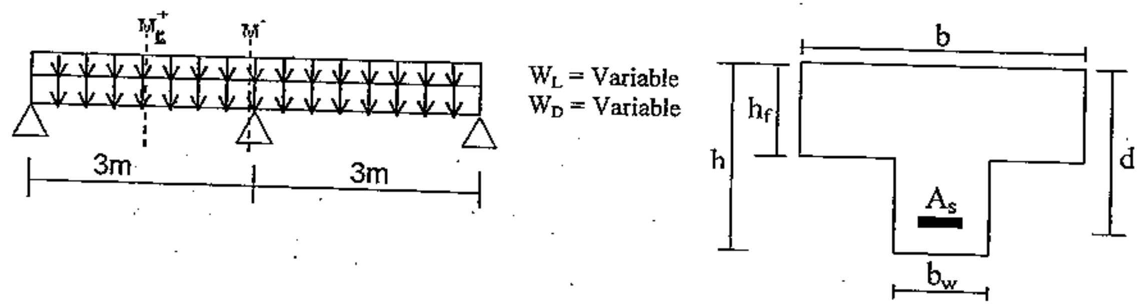
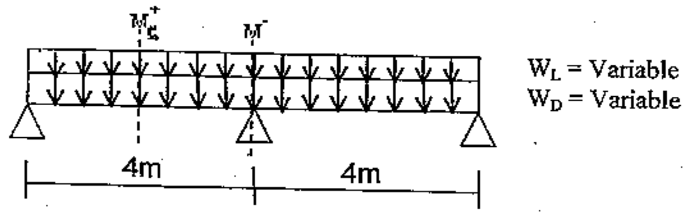


Table 4.2-a Parameters used in study number one (case 4)
2-span continuous beam (T cross-section) L=3m

f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	h_f(mm)	b(mm)	b_w(mm)	d(mm)	h(mm)	M^- A_s(mm^2)	M^+ A_s(mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
28	420	0.02125	0.25	0.00531	80	300	250	370	445	491	240	10	25	38.6
			0.5	0.01063				265	340	704	330			38.0
			0.75	0.01594				220	295	877	400			37.7
28	420	0.02125	0.25	0.00531	80	300	250	435	510	578	285	15	35	54.0
			0.5	0.01063				315	390	837	390			52.4
			0.75	0.01594				265	340	1056	470			53.0
28	420	0.02125	0.25	0.00531	80	300	250	495	570	657	325	20	45	69.4
			0.5	0.01063				360	435	956	445			68.6
			0.75	0.01594				300	375	1195	530			68.2

1 Includes own weight of beam



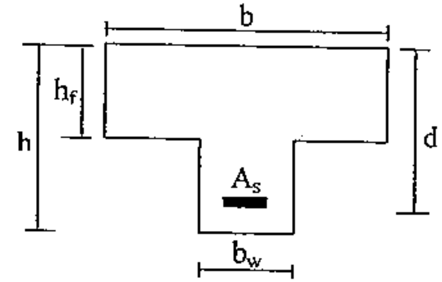


Table 4.2-b Parameters used in study number one (case 5)
2-span continuous beam (T cross-section) L=4m

f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	h_f(mm)	b(mm)	b_w(mm)	d(mm)	h(mm)	M^- A_s(mm^2)	M^+ A_s(mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
28	420	0.02125	0.25	0.00531	80	350	300	455	530	725	360	10	25	40.2
			0.5	0.01063				330	405	1052	485			39.3
			0.75	0.01594				275	350	1315	580			38.9
28	420	0.02125	0.25	0.00531	80	350	300	540	615	861	420	15	35	55.8
			0.5	0.01063				390	465	1243	570			54.7
			0.75	0.01594				325	400	1554	690			54.2
28	420	0.02125	0.25	0.00531	80	350	300	610	685	972	480	20	45	71.3
			0.5	0.01063				440	515	1403	650			70.1
			0.75	0.01594				370	445	1769	780			69.5

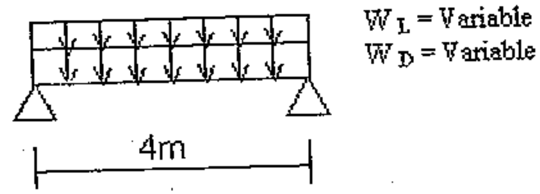
1 Includes own weight of beam

4.4.2 Study number two

The purpose of this study is to detect the immediate deflection behavior associated with the following:

- Simply supported beam
- Tension steel content $\rho = (0.25, 0.5, 0.75)\ \rho_{max}$.
- Span length L = (4, 5) m
- Concrete compressive strength $f_c' =$ (21, 28, 35) MPa
- Steel yield strength $f_y =$ (300, 420) MPa
- Uniformly distributed dead load = (10, 15) kN/m
- Uniformly distributed live load = (25, 35) kN/m
- Four cases in this study, the first two cases for rectangular cross-section and the remaining cases for T cross-section

Tables (4.3-a to 4.4-b) illustrate the variables used in this study.



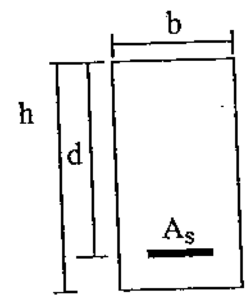
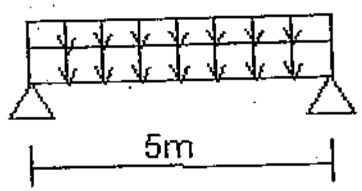


Table 4.3-a Parameters used in study number two (case 1)
Simply supported beam (rectangular cross-section) L=4m

f'_c(MPa)	f_y(MPa)	β_1	ρ_{max}	ρ/ρ_{max}	ρ	b(mm)	d(mm)	h(mm)	A_s(mm²)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
21	300	0.85	0.02529	0.5	0.01264	300	355	430	1347	10	25	38.1
28		0.85	0.03372		0.01686		305	380	1543			37.7
35		0.81	0.04016		0.02008		280	355	1687			37.6
21	420	0.85	0.01594		0.00797		375	450	896			38.2
28		0.85	0.02125		0.01063		325	400	1036			37.9
35		0.81	0.02531		0.01266		295	370	1120			37.7
21	300	0.85	0.02529	0.5	0.01264	300	420	495	1593	15	35	53.6
28		0.85	0.03372		0.01686		365	440	1846			53.2
35		0.81	0.04016		0.02008		330	405	1988			52.9
21	420	0.85	0.01594		0.00797		445	520	1064			53.7
28		0.85	0.02125		0.01063		385	460	1227			53.3
35		0.81	0.02531		0.01266		350	425	1329			53.1

1 Includes own weight of beam



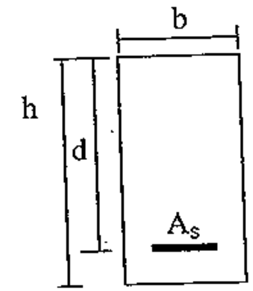
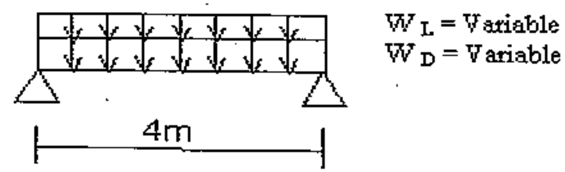


Table 4.3-b Parameters used in study number two (case 2)
Simply supported beam (rectangular cross-section) L=5m

f'_c(MPa)	f_y(MPa)	β_1	ρ_{max}	ρ/ρ_{max}	ρ	b(mm)	d(mm)	h(mm)	A_s(mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
21	300	0.85	0.02529	0.5	0.01264	300	445	520	1688	10	25	38.7
28		0.85	0.03372		0.01686		385	460	1947			38.3
35		0.81	0.04016		0.02008		350	425	2109			38.1
21	420	0.85	0.01594		0.00797		470	545	1124			38.9
28		0.85	0.02125		0.01063		405	480	1291			38.5
35		0.81	0.02531		0.01266		370	445	1405			38.2
21	300	0.85	0.02529	0.5	0.01264	300	530	605	2010	15	35	54.4
28		0.85	0.03372		0.01686		455	530	2301			53.8
35		0.81	0.04016		0.02008		415	490	2500			53.5
21	420	0.85	0.01594		0.00797		560	635	1339			54.6
28		0.85	0.02125		0.01063		480	555	1530			54.0
35		0.81	0.02531		0.01266		440	515	1671			53.7

1 Includes own weight of beam



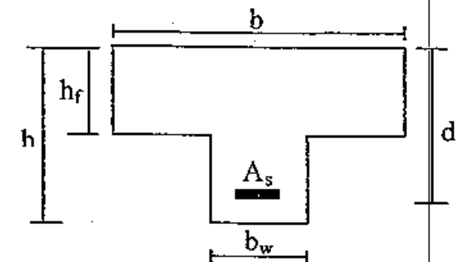


Table 4.4-a Parameters used in study number two (case 3)

Simply supported beam beam (T cross-section) L=4m

f'_c(MPa)	f_y(MPa)	β_1	ρ_{max}	ρ/ρ_{max}	ρ	b(mm)	b_w(mm)	d(mm)	h(mm)	A_s(mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
21	300	0.85	0.02529	0.5	0.01264	1000	200	195	270	2466	10	25	37.8
28		0.85	0.03372		0.01686			170	245	2866			37.7
35		0.81	0.04016		0.02008			155	230	3113			37.6
21	420	0.85	0.01594		0.00797			205	280	1634			37.9
28		0.85	0.02125		0.01063			180	255	1913			37.8
35		0.81	0.02531		0.01266			165	240	2088			37.7

1 Includes own weight of beam

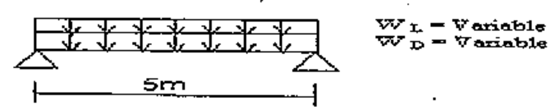


Table 4.4-b Parameters used in study number two (case 4)

Simply supported beam beam (T cross-section) L=5m

f'_c(MPa)	f_y(MPa)	β_1	ρ_{max}	ρ/ρ_{max}	ρ	b(mm)	b_w(mm)	d(mm)	h(mm)	A_s(mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
21	300	0.85	0.02529	0.5	0.01264	1250	200	220	295	3477	10	25	38.4
28		0.85	0.03372		0.01686			190	265	4004			38.3
35		0.81	0.04016		0.02008			175	250	4393			38.2
21	420	0.85	0.01594		0.00797			230	305	2291			38.5
28		0.85	0.02125		0.01063			200	275	2656			38.3
35		0.81	0.02531		0.01266			185	260	2927			38.3

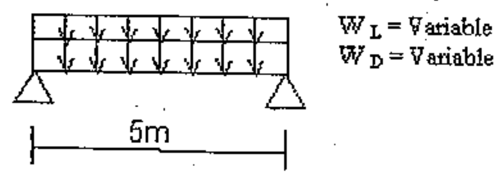
1 Includes own weight of beam

4.4.3 Study number three

This study has illustrated the effects on immediate deflection by using the following variables:

- Beam Types (simply supported, 3-span continuous, cantilever)
- Tension steel ratio $\rho = (0.25, 0.5, 0.75)\ \rho_{max}$
- Compression steel ratio $\rho' = (0.25, 0.5, 0.75)\ \rho$
- Span length L = 5m for simply supported and 3-span continuous beams, L = 3m for cantiliver bear
- Concrete compressive strength $f_c' = 28$ MPa
- Steel yield strength $f_y = 420$ MPa
- Uniformly distributed dead load = (10, 15) kN/m
- Uniformly distributed live load = (25, 35) kN/m
- Six cases in this study, the first three cases for rectangular cross-section and the remaining cases for T cross-section

Tables (4.5-a to 4.6-c) show the variables used in this study.



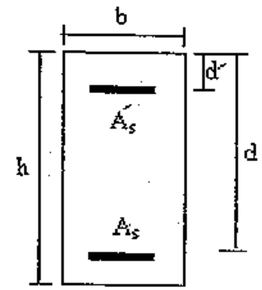
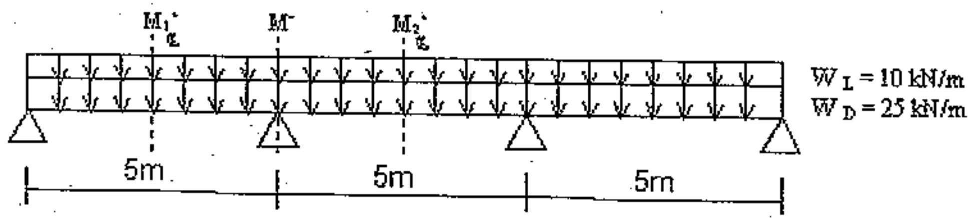


Table 4.5-a Parameters used in study number three (case 1)
Simply supported beam (rectangular cross-section) L=5m

f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	b(mm)	d(mm)	h(mm)	A_s(mm^2)	A'_s(mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
28	420	0.0213	0.5	0.01063	0.25	0.00266	300	405	480	1291	323	10	25	38.5
					0.5	0.00531					645			
					0.75	0.00797					968			
			0.75	0.01594	0.25	0.00398		335	410	1602	400			38.0
					0.5	0.00797					801			
					0.75	0.01195					1201			
28	420	0.0213	0.5	0.01063	0.25	0.00266	300	480	555	1530	383	15	35	54.0
					0.5	0.00531					765			
					0.75	0.00797					1148			
			0.75	0.01594	0.25	0.00398		395	470	1889	472			53.4
					0.5	0.00797					944			
					0.75	0.01195					1416			

1 Includes own weight of beam



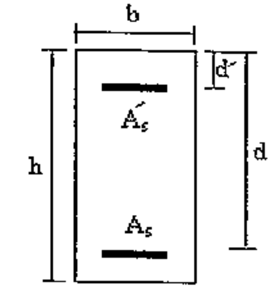


Table 4.5-b Parameters used in study number three (case 2)

3-span continuous beam (rectangular cross-section) L=5m

										M^-		M^+ (Exterior spans)		M^+ (Interior span)			
f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	b(mm)	d(mm)	h(mm)	A_s(mm²)	A'_s(mm²)	A_s(mm²)	A'_s(mm²)	A_s(mm²)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
28	420	0.0213	0.5	0.0106	0.25	0.00266	300	360	435	1148	287	870	400	270	10	25	38.1
					0.5	0.00531					574						
					0.75	0.00797					861						
			0.75	0.0159	0.25	0.00398		300	375	1434	359	1060	550	330			37.7
					0.5	0.00797					717						
					0.75	0.01195					1076						
28	420	0.0213	0.5	0.0106	0.25	0.00266	300	430	505	1371	343	1020	550	430	15	35	53.6
					0.5	0.00531					685						
					0.75	0.00797					1028						
			0.75	0.0159	0.25	0.00398		355	430	1697	424	1250	600	390			53.1
					0.5	0.00797					849						
					0.75	0.01195					1273						

1 Includes own weight of beam

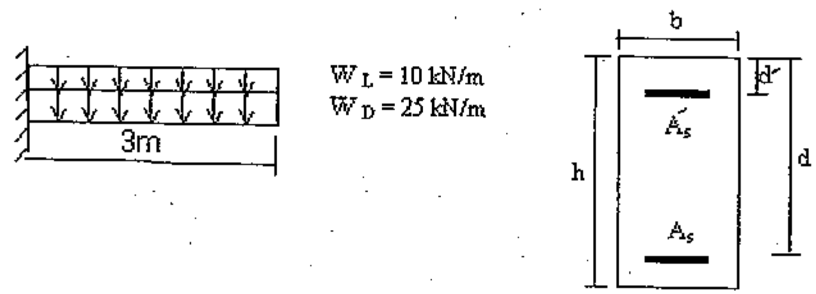
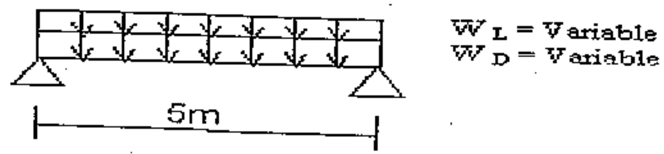


Table 4.5-c Parameters used in study number three (case 3)

Cantilever beam (rectangular cross-section) L=3m

f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	b(mm)	d(mm)	h(mm)	A_s(mm^2)	A'_s(mm^2)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
28	420	0.02	0.5	0.011	0.25	0	300	490	565	1562	390	10	25	39.1
					0.5	0.01					781			
					0.75	0.01					1171			
			0.75	0.016	0.25	0	300	405	480	1936	484			38.5
					0.5	0.01					968			
					0.75	0.01					1452			
28	420	0.02	0.5	0.011	0.25	0	300	575	650	1833	458	15	35	54.7
					0.5	0.01					916			
					0.75	0.01					1375			
			0.75	0.016	0.25	0	300	475	550	2271	568			54.0
					0.5	0.01					1136			
					0.75	0.01					1703			

1 Includes own weight of beam



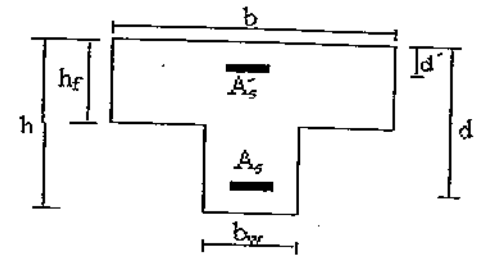


Table 4.6-a Parameters used in study number three (case 4)
Simply supported beam (T cross-section) with compression steel L=5m

f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	b(mm)	b_w(mm)	d(mm)	h(mm)	A_s(mm²)	A'_s(mm²)	w_L(kN/m)	w_D(kN/m)	w_a[1](kN/m)
28	420	0.021	0.5	0.01063	0.25	0.00266	1250	200	200	275	2656	664	10	25	38.3
					0.5	0.00531						1328			
					0.75	0.00797						1992			
			0.75	0.01594	0.25	0.00398			165	240	3287	822			38.1
					0.5	0.00797						1644			
					0.75	0.01195						2465			

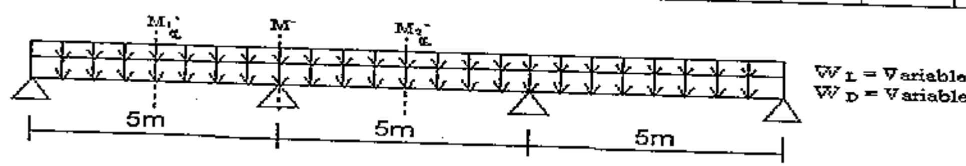
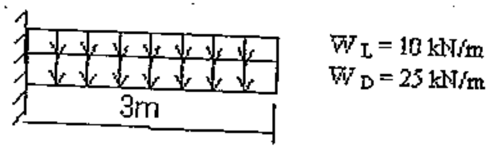


Table 4.6-b Parameters used in study number three (case 5)
3-span continuous beam (T cross-section) with compression steel L=5m

f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	b(mm)	b_w(mm)	d(mm)	h(mm)	M^- A_s(mm²)	M^- A'_s(mm²)	M^+ Exterior A_s(mm²)	M^+ Interior A_s(mm²)	w_a[1](kN/m)
28	420	0.021	0.5	0.0106	0.25	0.00266	550	250	400	475	1063	266	740	740	39.8
					0.5	0.00531	550	250				531			
					0.75	0.00797	550	250				797			
			0.75	0.0159	0.25	0.00398	550	250	330	405	877	329	910	605	39.4
					0.5	0.00797	550	250				657			
					0.75	0.01195	550	250				986			

1 Includes own weight of beam



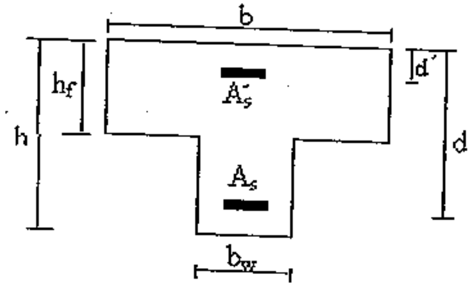


Table 4.6-c Parameters used in study number three (case 6)
Cantilever beam (T cross-section) with compression steel L=3m

f'_c(MPa)	f_y(MPa)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	b(mm)	b_w(mm)	d(mm)	h(mm)	$A_s(mm^2)$	$A'_s(mm^2)$	w_a[1](kN/m)
28	420	0.02125	0.5	0.01063	0.25	0.003	750	250	515	590	1368	342	39.5
					0.5	0.005						684	
					0.75	0.008						1026	
			0.75	0.01594	0.25	0.004			425	500	1693	423	38.96
					0.5	0.008						847	
					0.75	0.012						1270	

1 Includes own weight of beam

4.4.4 Study number four

This study had an experimental results compared to immediate deflection values calculated by thesis approach and ACI 318M-99 provisions, by using the following:

- Simply supported beam
- Span length L = 2.5m
- Concrete compressive strength $f_c' = 31.2$ MPa
- Steel yield strength $f_y = 270$ MPa
- Variable uniform distributed load
- Square cross-section

Table (4.7) illustrates the variables used in this study.

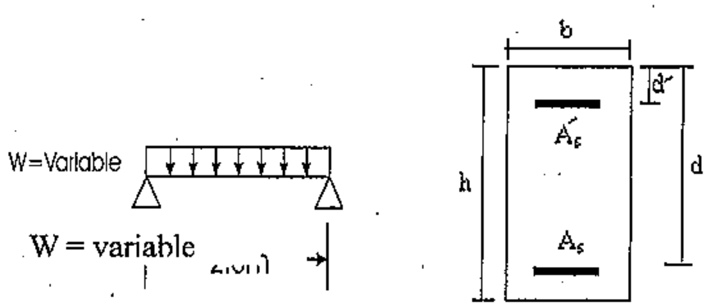


Table 4.7. Experimental data used in study number four

Simply supported beam (square cross-section) L=2.5m *

f'_c (MPa)	f_y (MPa)	b (mm)	d (mm)	h(mm)	A_s (mm^2)	A_s' (mm^2)
31.2	270	200	150	200	400	80

* (Ghali, 1993)

5.Results and Discussion

5.1 Introduction

Concrete deformation is probabilistic and our knowledge is imperfect to even provide mean value functions and variance. Calculations can, at best, provide a guide to probable actual deflections. This is because of the uncertainties regarding material properties, effects of cracking and load history for the member under consideration.

Because of these reasons, a method is proposed to give better precision in the calculations, as long as they give reasonable results compared with the experimental data and the ACI provisions.

5.2 Calculation of Deflection

The method of this thesis is used to calculate immediate deflection, by using variable moment of inertia across the beam and the results are compared with ACI provisions and experimental results.

5.3 Results

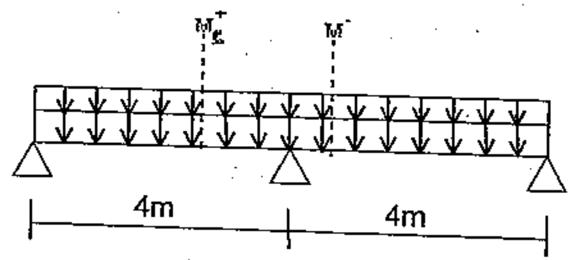
5.3.1 Results of Study number one

Two span continuous beams under uniform distributed load were studied, for both rectangular and T cross-sections the variables were (beam length, load level and tension steel ratio).

A rectangular cross-section is illustrated in Tables (5.1-a, 5.1-b, 5.1-c) which shows that the immediate deflections specified by the ACI provisions and thesis approach were almost identical, with the ACI provisions being conservative in all cases.

For T-sections, it is noticed from Tables (5.2-a, 5.2-b, 5.2-c) that the immediate deflections calculated by the ACI provisions were more conservative than thesis approach for all tension steel ratios ρ = (0.25, 0.5, 0.75)ρ_{max}.

Tables (5.1-a to 5.2-c) illustrate the results of study number one.



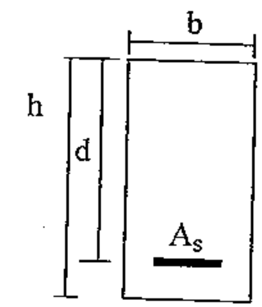


Table 5.1-b Results of study number one (case 2)

2- span continuous beam (rectangular cross-section) L=4m

			M^-	M^+				
ρ/ρ_{max}	M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.25	77.6	51.1	36.17	5.42	7.99	33.15	0.59	0.63
0.5	75.8	29.6	16.00	2.90	5.18	8.67	2.22	2.34
0.75	75.0	22.2	10.27	2.25	3.98	3.90	4.99	5.15
0.25	108.8	68.9	56.74	9.10	13.43	51.90	0.53	0.56
0.5	106.6	39.2	24.33	4.88	8.61	12.13	2.25	2.36
0.75	105.7	28.9	15.41	3.81	6.63	5.84	4.65	4.85
0.25	139.8	85.6	78.61	13.16	19.42	71.77	0.49	0.52
0.5	137.3	48.2	33.16	7.06	12.42	15.66	2.26	2.35
0.75	136.3	35.9	21.30	5.51	9.84	8.11	4.30	4.50

1 Deflection at midspan

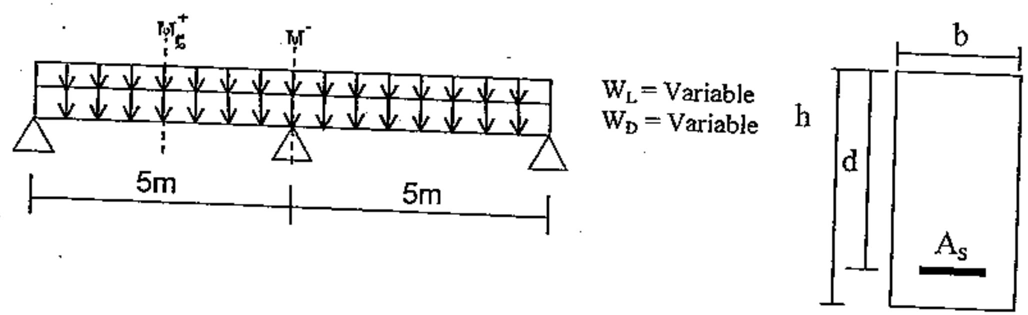


Table 5.1-c Results of study number one (case 3)

2- span continuous beam (rectangular cross-section) L=5m

			M⁻	M⁺				
ρ/ρ_{max}	M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.25	125.3	79.1	64.59	10.32	15.23	59.05	0.83	0.89
0.5	121.3	44.7	27.47	5.41	9.66	13.70	3.54	3.71
0.75	119.6	32.9	17.30	4.15	7.38	6.49	7.41	7.72
0.25	174.6	105.9	100.04	16.91	24.98	91.29	0.75	0.80
0.5	170.0	59.6	42.20	8.99	16.05	19.92	3.44	3.57
0.75	168.1	43.8	26.58	6.97	12.45	10.15	6.58	6.93
0.25	225.3	135.2	135.07	23.30	34.39	123.24	0.72	0.77
0.5	219.6	74.7	55.46	12.23	21.57	25.40	3.48	3.62
0.75	217.4	55.7	35.72	9.67	17.27	13.79	6.24	6.60

1 Deflection at midspan

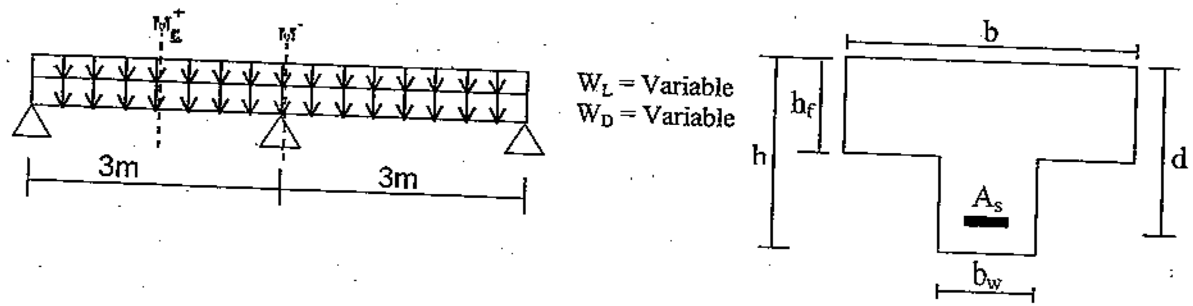
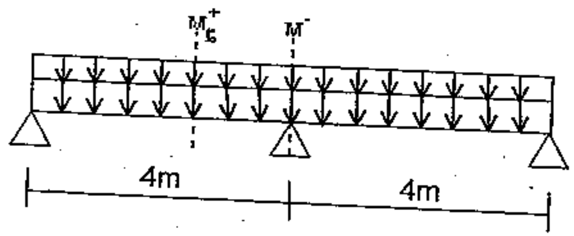


Table 5.2-a Results of study number one (case 4)
2-span continuous beam (T cross-section) L=3m

		M+			M-					
ρ/ρ_{max}	M_a(kN.m)	M_{cr}(kN.m)	I_g*10^8(mm^4)	I_{cr}*10^8(mm^4)	M_{cr}(kN.m)	I_g*10^8(mm^4)	I_{cr}*10^8(mm^4)	I_e*10^8(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.25	43.5	31.8	19.67	3.01	30.6	18.36	3.70	18.10	0.33	0.35
0.5	42.8	18.7	8.85	1.36	17.8	8.19	2.34	6.11	0.98	1.03
0.75	42.4	14.1	5.81	1.04	13.4	5.35	1.80	2.42	2.52	2.59
0.25	60.8	41.7	29.44	4.85	40.1	27.64	6.02	26.91	0.31	0.33
0.5	59.0	24.5	13.30	2.27	23.5	12.36	3.93	7.97	1.06	1.11
0.75	59.6	18.7	8.85	1.78	17.8	8.19	3.14	3.53	2.45	2.5
0.25	78.1	51.98	40.94	7.2	50.14	38.58	8.86	37.35	0.29	0.31
0.5	77.1	30.43	18.38	3.35	29.2	17.15	5.87	10.34	1.06	1.11
0.75	76.7	22.66	11.84	2.61	21.7	10.99	4.55	4.58	2.42	2.49

1 Deflection at midspan



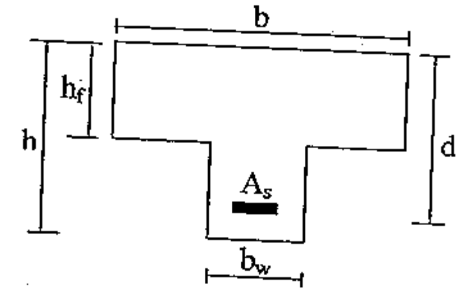
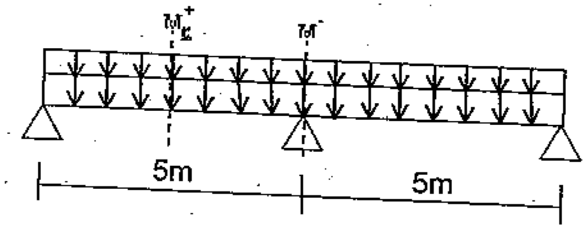


Table 5.2-b Results of study number one (case 5)
2-span continuous beam (T cross-section) L=4m

ρ/ρ_{max}	M_a(kN.m)	M^+ M_{cr}(kN.m)	M^+ $I_g*10^8(mm^4)$	M^+ $I_{cr}*10^8(mm^4)$	M^- M_{cr}(kN.m)	M^- $I_g*10^8(mm^4)$	M^- $I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.25	80.3	53.7	39.22	6.53	52.0	37.22	8.26	35.87	0.55	0.58
0.5	78.5	31.5	17.65	3.00	30.4	16.61	5.42	10.55	1.83	1.93
0.75	77.7	23.6	11.44	2.36	22.7	10.72	4.21	4.53	4.33	4.45
0.25	111.5	72.1	60.97	10.82	70.1	58.15	13.81	55.66	0.49	0.52
0.5	109.4	41.4	26.60	4.97	40.1	25.14	8.95	14.25	2.80	2.91
0.75	108.4	30.7	17.01	3.96	29.6	16.00	6.95	6.57	4.20	4.32
0.25	142.6	89.3	83.97	15.57	86.9	80.35	19.91	76.52	0.46	0.49
0.5	140.1	50.7	36.01	72.06	49.1	34.15	12.86	18.01	1.96	2.05
0.75	139.1	38.0	23.34	5.85	36.7	22.03	10.25	9.06	3.88	4.04

1 Deflection at midspan



W_L = Variable
W_D = Variable

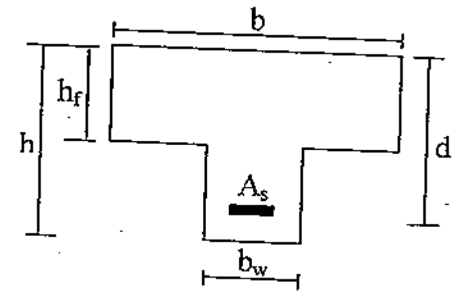


Table 5.2-c Results of study number one (case 6)
2-span continuous beam (T cross-section) L=5m

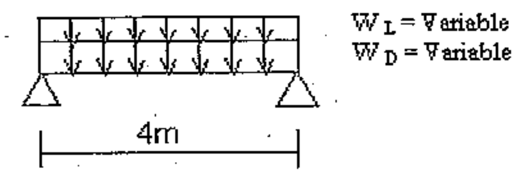
		M^+			M^-					
ρ/ρ_{max}	M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.25	130.9	83.8	70.67	12.36	81.73	67.84	16.11	64.62	0.76	0.81
0.5	126.9	47.08	29.82	5.52	45.72	28.39	10.05	15.9	3.21	3.02
0.75	125.1	34.78	18.96	4.28	33.71	17.97	7.74	7.23	6.75	6.95
0.25	180.3	111.45	108.32	18.88	108.92	104.39	26.2	98.82	0.7	0.74
0.5	175.6	62.38	45.43	9.42	60.69	43.42	16.59	22.67	3.02	3.15
0.75	173.6	46.07	28.87	7.31	44.73	27.47	12.95	11.16	6.06	6.32
0.25	230.6	170.08	199.4	21.4	137.06	137.83	35.17	178	0.68	0.73
0.5	224.9	77.87	59.22	12.97	76.06	56.98	22.26	28.49	3.1	3.23
0.75	222.5	57.15	37.27	10.1	55.71	35.72	17.27	14.52	5.99	6.28

1 Deflection at midspan

5.3.2 Results of Study number two

Simply supported beams under uniform distributed load were studied for both rectangular and T cross-sections. The variable parameters were (beam length, load level, concrete compression strength and steel yield strength). For a rectangular cross-section, see Tables (5.3-a, 5.3-b), the values of immediate deflections calculated by the ACI provisions were larger by a small amount than the values of thesis approach. For T cross-sections, see Tables (5.4-a, 5.4-b), the ACI provisions with regard to immediate deflections and thesis approach give almost identical results for all cases.

Tables (5.3-a to 5.4-b) show the results of study number two.



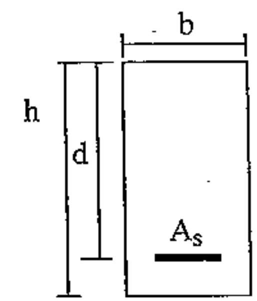
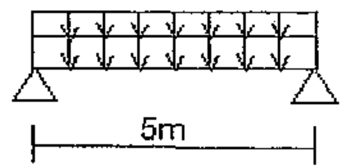


Table 5.3-a Results of study number two (case 1)
Simply supported beam (rectangular cross-section) L=4m

M_a(kN.m)	M_{cr}(kN.m)	I_g*10^8(mm^4)	$I_{cr}*10^8$(mm^4)	I_e*10^8(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)
76.2	29.7	19.88	8.51	9.18	6.37	6.42
75.5	26.7	13.72	5.97	6.31	7.96	8.01
75.1	26.1	11.18	4.82	5.09	8.79	8.84
76.5	32.5	22.78	7.14	8.34	7.01	7.10
75.8	29.6	16.00	5.18	5.83	8.62	8.71
75.3	28.3	12.66	4.06	4.52	9.88	9.98
107.1	39.3	30.32	14.10	14.90	5.53	5.56
106.3	35.9	21.30	10.23	10.66	6.66	6.69
105.8	34.0	16.61	7.90	7.90	7.99	8.03
107.5	43.4	35.15	11.93	13.46	6.12	6.18
106.6	39.2	24.33	8.61	9.39	7.55	7.60
106.1	37.4	19.19	6.78	7.33	8.62	8.68

1 Deflection at midspan



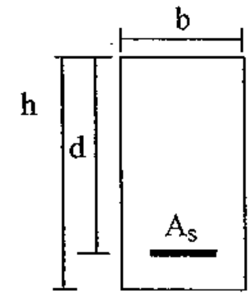
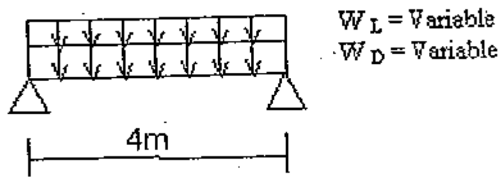


Table 5.3-b Results of study number two (case 2)
Simply supported beam (rectangular cross-section) L=5m

M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
121.1	43.4	35.15	16.77	17.61	8.26	8.31
119.7	39.2	24.33	12.01	12.01	10.39	10.44
118.9	37.4	19.19	9.42	9.42	11.77	11.82
121.6	47.6	40.47	14.06	15.64	9.31	9.40
120.2	42.7	27.65	10.03	10.82	11.55	11.63
119.4	41.0	22.03	8.01	8.58	12.96	13.03
169.9	58.7	55.36	28.33	29.44	6.94	6.97
168.2	52.0	37.22	19.82	19.82	8.86	8.88
167.3	49.7	29.41	15.71	15.71	9.94	9.97
170.5	64.7	64.01	23.78	25.97	7.86	7.94
168.7	57.1	42.74	16.69	17.70	9.93	9.98
167.8	54.9	34.15	13.48	13.48	11.60	11.66

1 Deflection at midspan



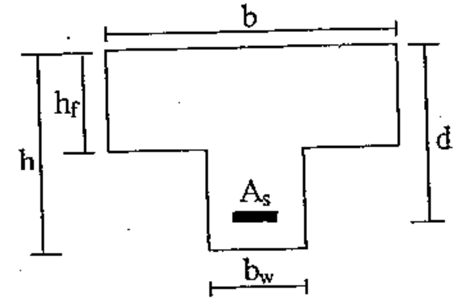


Table 5.4-a Results of study number two (case 3)

Simply supported beam beam (T cross-section) L=4m

M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
75.7	10.8	6.26	4.70	4.70	12.45	12.45
75.4	10.3	4.68	3.45	3.45	14.66	14.66
75.3	10.1	3.87	2.73	2.73	16.53	16.53
75.8	11.6	6.99	3.89	3.89	15.07	15.07
75.5	11.1	5.28	2.94	2.94	17.22	17.22
75.4	11.0	4.40	2.38	2.38	19.03	19.03

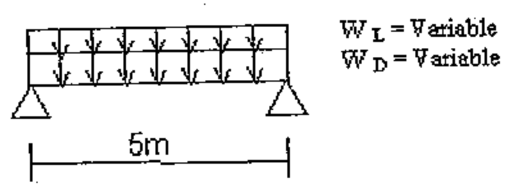


Table 5.4-b Results of study number two (case 4)

Simply supported beam beam (T cross-section) L=5m

M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$Icr*10^8(mm^4)$	$Ie*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
120.1	13.3	8.73	8.44	8.44	17.20	17.20
119.7	12.4	6.33	6.01	6.01	20.83	20.83
119.4	12.4	5.32	4.91	4.91	22.79	22.79
120.3	14.2	9.65	6.86	6.86	21.17	21.18
119.8	13.3	7.07	5.03	5.03	24.92	24.92
119.6	13.3	5.98	4.18	4.18	26.81	26.81

1 Deflection at midspan

5.3.3 Results of Study number three

Three types of beams were used in this study under uniform distributed load, for both rectangular and T cross-sections. The variable used were (tension steel ratio, compression steel ratio, load level and types of beam).

For a rectangular cross-section, see Tables (5.5-a, 5.5-b, 5.5-c), case 1 studied for a 5m simply supported beam. The values of immediate deflections determined by the ACI provisions and thesis approach were very close in all cases with somewhat larger values in the ACI provisions.

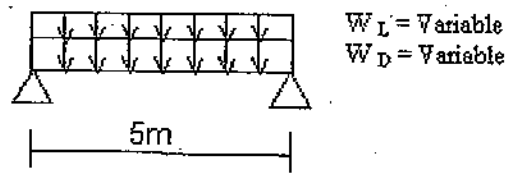
Case 2 studied three equal-span, continuous beam, with a span length of 5m. For the exterior spans, the ACI provisions were more conservative with regard to immediate deflections than the thesis approach for the case of tension steel ratio $\rho = 0.5\rho_{max}$. They were unconservative for the case of tension steel ratio $\rho = 0.75\rho_{max}$. For the midspan the values of immediate deflections by the ACI provisions and thesis approach were very close in all cases.

Case 3 studied a 3m cantilever beam. The results showed that the ACI provisions were more conservative than thesis approach with regard to the immediate deflections.

For T cross-section, see Tables (5.6-a, 5.6-b, 5.6-c), case 4 was studied for a 5m simply supported beam under uniform distributed load, with one load level. The results of immediate deflections calculated by the ACI provisions and thesis approach were almost identical in all cases.

Case 5 studied three equal-span, continuous beam, with a span length of 5m. As in case 2, for the exterior spans, the ACI provisions were more conservative with regard to immediate deflections than the thesis approach for the case of tension steel ratio $\rho = 0.5\rho_{max}$. They were unconservative for the case of tension steel ratio $\rho = 0.75\rho_{max}$. For the midspan the values of immediate deflections by the ACI provisions and thesis approach were very close in all cases.

Case 6 studied a 3m cantilever beam. As in case 3, the results showed that the ACI provisions were more conservative than thesis approach with regard to immediate deflections.



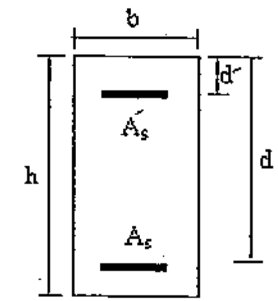
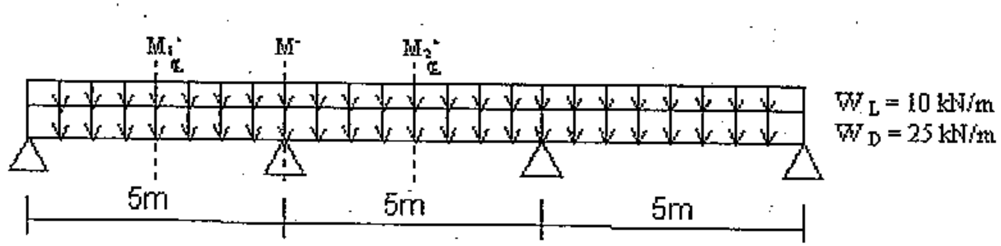


Table 5.5-a Results of study number three (case 1)
Simply supported beam (rectangular cross-section) with compression steel L=5m

p/p_{max}	M_a(kN.m)	M_{cr}(kN.m)	I_g*10^8(mm^4)	I_{cr}*10^8(mm^4)	I_e*10^8(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.5	120.2	42.7	27.65	10.21	10.99	11.36	11.45
				10.38	11.15	11.20	11.28
				10.53	11.30	11.06	11.14
0.75	118.6	31.1	17.23	7.81	7.81	15.86	15.89
				7.99	7.99	15.50	15.53
				8.15	8.15	15.19	15.23
0.5	168.7	57.1	42.74	17.06	18.05	9.74	9.79
				17.39	18.37	9.57	9.62
				17.69	18.66	9.42	9.47
0.75	166.8	40.9	25.96	12.87	12.87	13.54	13.57
				13.22	13.22	13.18	13.21
				13.54	13.54	12.87	12.90

1 Deflection at midspan



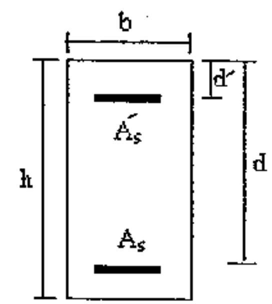
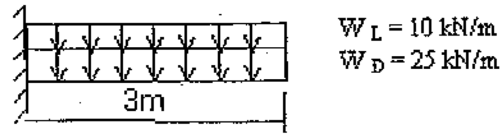


Table 5.5-b Results of study number three (case 2)
3-span continuous beam (rectangular cross-section) with compression steel L=5m

		M^-	M^+ (Exterior spans)				M^+ (Interior span)			
M_{cr}(kN.m)	I_g*10^8(mm^4)	I_{cr}*10^8(mm^4)	I_{cr}*10^8(mm^4)	I_e*10^8(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)	I_{cr}*10^8(mm^4)	I_e*10^8(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)
35.0	20.58	7.15	5.80	7.58	7.95	8.12	2.16	16.75	0.57	0.60
		7.25		7.60	7.93	8.10		16.78	0.56	0.59
		7.50		7.61	7.92	8.09		16.80	0.56	0.59
26.0	13.18	5.59	4.52	5.05	12.18	12.07	1.74	10.91	0.55	0.58
		5.70		5.07	12.12	12.03		10.94	0.55	0.57
		5.80		5.08	12.08	12.00		10.97	0.54	0.57
47.2	32.20	12.24	9.83	12.29	6.95	7.06	4.73	26.47	0.47	0.52
		12.45		12.32	6.93	7.04		26.53	0.47	0.52
		12.64		12.35	6.91	7.02		26.59	0.46	0.51
34.2	19.88	9.31	7.51	8.21	10.90	10.48	2.88	16.71	0.45	0.48
		9.54		8.24	10.85	10.44		16.78	0.45	0.47
		9.75		8.27	10.81	10.40		16.84	0.44	0.47

1 Deflection at midspan



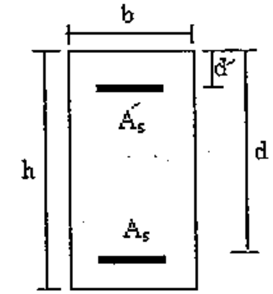
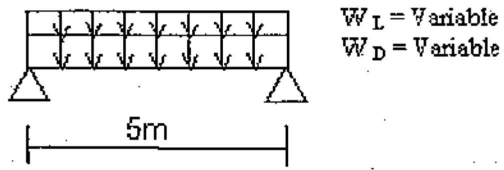


Table 5.5-c Results of study number three (case 3)

Cantilever (rectangular cross-section) with compression steel L=3m

p/p_{max}	M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.5	175.8	59.1	45.09	18.15	19.18	7.75	8.29
				18.51	19.52	7.62	8.15
				18.84	19.83	7.51	8.02
0.75	173.1	42.7	27.65	13.88	13.88	10.93	11.28
				14.27	14.27	10.64	10.97
				14.62	14.62	10.39	10.71
0.5	246.1	78.3	68.66	29.42	29.42	7.14	7.57
				30.07	30.07	6.99	7.40
				30.67	30.67	6.86	7.26
0.75	242.8	56.0	41.59	22.50	22.50	9.53	9.76
				23.21	23.21	9.24	9.46
				23.85	23.85	9.00	9.21

1 Deflection at midspan



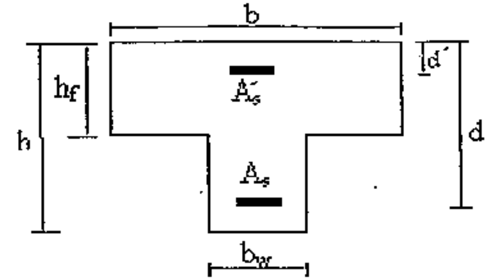


Table 5.6-a Results of study number three (case 4)
Simply supported beam (T cross-section)with compression steel L=5m

M_a(kN.m)	M_{cr}(kN.m)	I_g10^8(mm^4)	$I_{cr}*10^8$(mm^4)	Ie*10^8(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)
119.8	13.3	7.07	5.05	5.05	24.85	24.85
			5.06	5.06	24.78	24.78
			5.07	5.07	24.71	24.72
119.2	10.2	4.71	3.80	3.80	32.84	32.84
			3.82	3.82	32.73	32.73
			3.83	3.83	32.63	32.63

1 Deflection at midspan

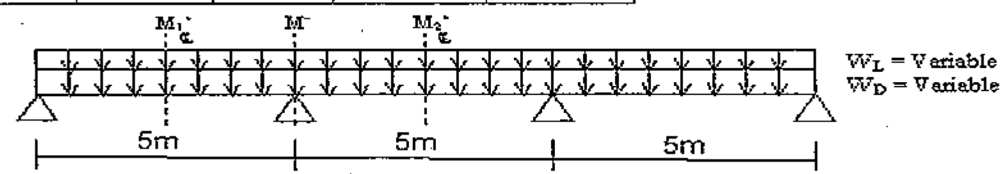
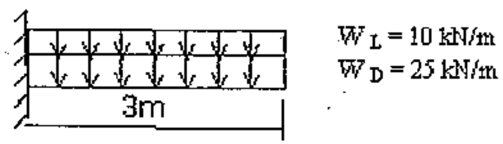


Table 5.6-b Parameters used in study number three (case 5)
3-span continuous beam (T cross-section) with compression steel L=5m

		M +		M$^-$			(Exterior spans)				(Interior span)			
ρ/ρ_{max}	M_a(kN.m)	M_{cr}(kN.m)	Ig*10^8(mm^4)	M_{cr}(kN.m)	Ig*10^8(mm^4)	$I_{cr}*10^8$(mm^4)	$I_{cr}*10^8$(mm^4)	I_e(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)	$I_{cr}*10^8$(mm^4)	I_e(mm^4)	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.5	99.4	41.4	30.20	34.8	22.30	8.42	7.03	11.07	5.43	5.61	7.03	23.89	0.34	0.37
						8.75		11.12	5.41	5.59		23.99	0.33	0.37
						9.05		11.17	5.39	5.56		24.08	0.33	0.37
0.75	98.4	30.3	19.10	25.3	13.84	6.48	5.51	6.54	9.50	9.36	5.51	15.31	0.30	0.32
						6.85		6.60	9.39	9.28		15.42	0.30	0.32
						7.19		6.65	9.30	9.21		15.52	0.29	0.32

1 Deflection at midspan



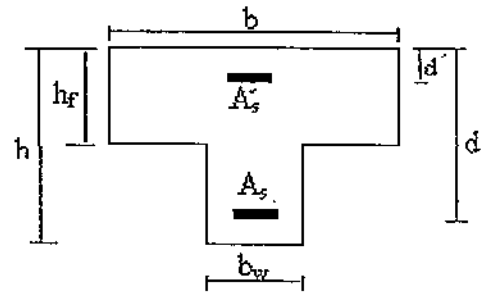


Table 5.6-c Parameters used in study number three (case 6)
Cantilever beam (T cross-section) with compression steel L=3m

p/p_{max}	M_a(kN.m)	M_{cr}(kN.m)	$I_g*10^8(mm^4)$	$I_{cr}*10^8(mm^4)$	$I_e*10^8(mm^4)$	Def.Thesis[1](mm)	Def.ACI[1](mm)
0.5	177.75	53.72	42.79	17.58	17.58	8.65	9.15
				17.94	17.94	8.49	8.96
				18.27	18.27	8.34	8.8
0.75	175.32	38.58	26.04	13.39	13.39	11.57	11.85
				13.78	13.78	11.24	11.51
				14.14	14.14	10.97	11.22

1 Deflection at midspan

5.3.4 Results of Study number four

A simply supported beam, under uniform load, having a 2.5m span, was used in this study, see Table (5.7), and compared with the results of an rxperimental study by Ghali, 1993. The variable used was load level. It was noticed that ACI provisions were more conservative than thesis approach with regard to immediate deflections. Both of them have larger values of immediate deflections than the experimental results. In other words, the ACI provisions and thesis approach were conservative with regard to immediate deflections. Table (5.7) shows the results of study Number Four.

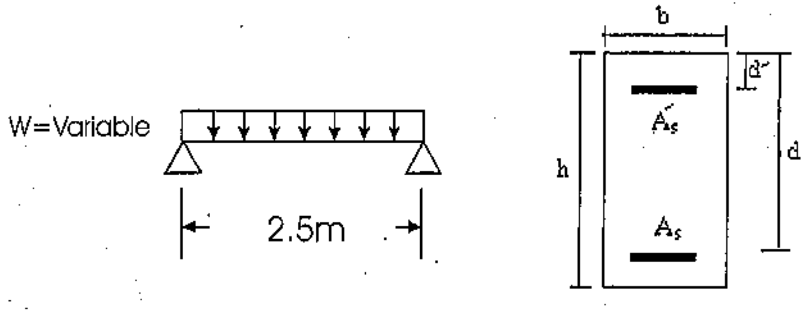


Table 5.7. Results of study number four
Simply supported beam-comparison of experimental results with ACI and thesis (L=2.5m)

M_{cr}(kN/m)	M_a/M_{cr}	Def.Thesis(mm)	Def.ACI(mm)	Def.Exp.(mm)
5.2	1.11	1.22	1.32	1.30
	1.36	2.15	2.29	2.10
	1.60	3.20	3.33	2.90
	1.98	4.90	5.01	4.30
	2.47	7.09	7.10	5.90
	2.72	8.02	8.09	6.80
	3.21	10.62	10.68	8.40
	3.70	12.31	12.34	10.10

5.4 Discussion of Results

The following is observed

1. Simply supported beam

   a) Rectangular cross-section

There are some differences between the ACI provisions and thesis approach with regard to immediate deflections. The ACI provisions were more conservative than thesis approach in all cases. See Tables(5.3-a, 5.3-b, 5.5-a).

   b) T cross-section:

The values of immediate deflections calculated by ACI provisions and thesis approach were almost identical in all cases. See Tables (5.4-a, 5.4-b, 5.6-a).

For both rectangular and T cross-sections, it was noticed that the beam may be divided into two zones.

Zone 1- $M_a/M_{cr} < 1$, so that I_g was used.

Zone 2- $1 \leq M_a/M_{cr} \leq 3$, So that I_e was used.

This division according to thesis approach gave smaller immediate deflections than ACI provisions.

2. Two span continuous beam:

   a) Rectangular cross-section

The ACI provisions were more conservative than thesis approach with regard to immediate deflections. It was noticed that the beam may be divided into two zones.

Zone 1- $M_a/M_{cr} < 1$, so that I_g was used.

Zone 2- $1 \leq M_a/M_{cr} \leq 3$, So that I_e was used.

This division according to thesis approach gave smaller immediate deflections than ACI provisions. See Tables (5.1-a, 5.1-b, 5.1-c).

b) T cross-section

Immediate deflections calculated by ACI provisions were more conservative than thesis approach in all cases of $\rho = (0.25, 0.5, 0.75)\rho_{max}$. See Tables (5.2-a, 5.2-b, 5.2-c).

It was noticed that the beam may be divided into two zones.

Zone 1- $M_a/M_{cr} < 1$, so that I_g was used.

Zone 2- $1 \leq M_a/M_{cr} \leq 3$, So that I_e was used.

This division according to thesis approach gave smaller immediate deflections than ACI provisions.

3. Three span continuous beam:

The results of rectangular and T-cross-sections had the same trend. ACI provisions for deflections were more conservative in the case of $\rho = 0.5\rho_{max}$, and unconservative in the case of $\rho = 0.75\rho_{max}$. See Tables (5.5-b, 5.6-b).

T cross-sections in continuous beams have a peculiar behavior. They behave as T-sections in positive moment regions and as rectangular in negative moment regions. As a result, continuous beam have two cracking moment values. In all cases of this study, it was observed that the cracking moment of a T-sections was larger than the cracking moment of a rectangular section.

For positive moment regions, the beam was divided into two zones.

Zone 1- $M_a/M_{cr} < 1$, so that I_g was used.

6.Conclusions and Recommendations

6.1 Conclusions

It is normal to expect differences between the results obtained from ACI provisions and thesis approach because they use different procedures.

6.1.1 Simply supported beams

The deflections of ACI and thesis were very close for rectangular sections and almost identical for T-sections.

6.1.2 Two span continuous beams

The deflections of thesis were smaller than ACI values for all cases of rectangular and T-sections.

6.1.3 Three span continuous beams

The deflections of thesis were smaller than ACI values for all cases of rectangular and T-sections, except when $\rho = 0.75\rho_{max}$ where thesis deflections were larger than ACI, for rectangular or T-sections.

6.1.4 Cantilever beams

The deflections of thesis were smaller than ACI values for all cases of rectangular or T-sections. The difference between thesis and ACI was more pronounced than cases of simply supported, two or three span continuous beams.

6.2 General Observations

1. An increase in tension steel ratio decreases the depth of the cross-section consequently increasing the deflections.

2. An increase in compression steel ratio slightly decreases the deflections.

3. An increase in steel yield strength generally increases the deflections.

4. A change in concrete compressive strength has an insignificant effect on deflections.

5. An increase in span length increases the deflections.

6.3 Recommendations

- It is recommended that more studies be done considering cases of concentrated loads, non uniform loads and other load possibilities.
- It is important to do more studies to verify the cases where ACI values were unconservative compared to thesis . Those cases which were repeatedly checked but showed this trend warrant some attention.
- It is recommended that thesis approach be modified to include long-term deflections and the results compared to ACI.

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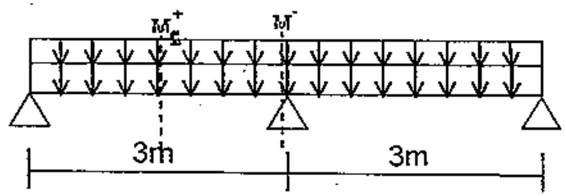
Appendix A : Results of cross-section design



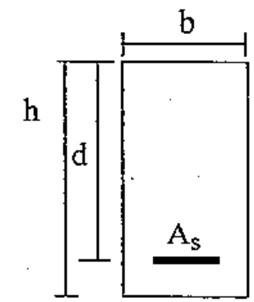
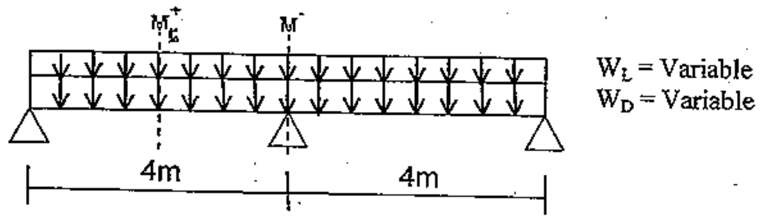


Table A.1-a Design of rectangular cross-section and reinforcement of study number one (case 1)
2- span continuous beam (L=3m)

				M^-					M^+			
b(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)
300	335	410	0.02125	0.25	0.00531	534	64.5	63.1	0.00333	335	41.0	31.6
	245	320		0.5	0.01063	781	65.4	62.1	0.00497	365	32.4	31.1
	200	275		0.75	0.01594	956	62.1	61.6	0.00758	455	32.1	30.8
300	400	475	0.02125	0.25	0.00531	638	91.6	89.2	0.00333	400	58.7	44.6
	290	365		0.5	0.01063	924	91.9	88.0	0.00506	440	46.0	44.0
	240	315		0.75	0.01594	1148	89.5	87.4	0.00750	540	45.8	43.7
300	450	525	0.02125	0.25	0.00531	717	116.3	115.1	0.00333	450	74.3	57.5
	325	400		0.5	0.01063	1036	115.5	113.7	0.00513	500	58.8	56.8
	275	350		0.75	0.01594	1315	115.7	113.1	0.00727	600	58.5	56.5



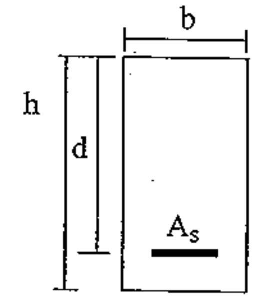
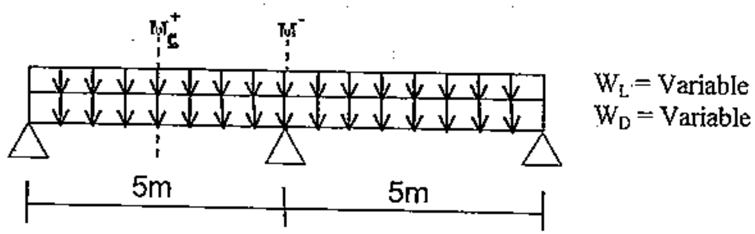


Table A.1-b Design of rectangular cross-section and reinforcement of study number one (case 2)
2- span continuous beam (L=4m)

				M^-					M^+			
b(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)
300	450	525	0.02125	0.25	0.00531	717	116.2	114.6	0.00333	450	74.4	57.3
	325	400		0.5	0.01063	1036	115.3	112.1	0.00508	495	58.0	56.0
	270	345		0.75	0.01594	1291	113.3	111.0	0.00741	600	57.3	55.5
300	535	610	0.02125	0.25	0.00531	853	164.3	161.3	0.00333	535	104.9	80.6
	385	460		0.5	0.01063	1227	162.0	158.3	0.00515	595	82.5	79.1
	320	395		0.75	0.01594	1530	159.0	157.0	0.00755	725	81.7	78.5
300	605	680	0.02125	0.25	0.00531	964	210.1	207.7	0.00333	605	134.7	103.9
	435	510		0.5	0.01063	1387	206.7	204.3	0.00517	675	105.9	102.1
	365	440		0.75	0.01594	1745	206.8	202.9	0.00731	800	103.5	101.4



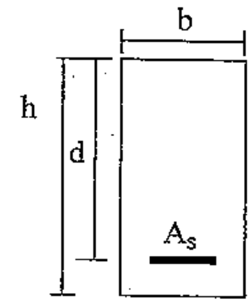
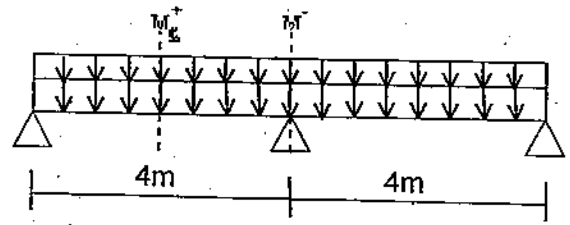


Table A.1-c Design of rectangular cross-section and reinforcement of study number one (case 3)
2- span continuous beam (L=5m)

				M^-					M^+			
b(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)
350	530	605	0.02125	0.25	0.00531	985	188.0	184.7	0.00333	618	120.1	92.4
	380	455		0.5	0.01063	1413	183.9	179.2	0.00508	675	92.5	89.6
	315	390		0.75	0.01594	1757	179.8	176.8	0.00735	810	90.3	88.4
350	625	700	0.02125	0.25	0.00531	1162	261.9	258.5	0.00333	728	167.0	129.3
	450	525		0.5	0.01063	1673	257.8	252.1	0.00508	800	130.1	126.1
	375	450		0.75	0.01594	2092	254.8	249.4	0.00731	960	127.4	124.7
400	665	740	0.02125	0.25	0.00531	1413	338.8	334.2	0.00333	886	216.2	167.1
	475	550		0.5	0.01063	2019	328.3	326.2	0.00516	980	167.7	163.1
	400	475		0.75	0.01594	2550	331.3	323.1	0.00731	1170	165.4	161.5



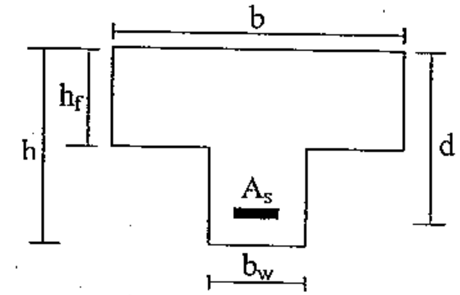
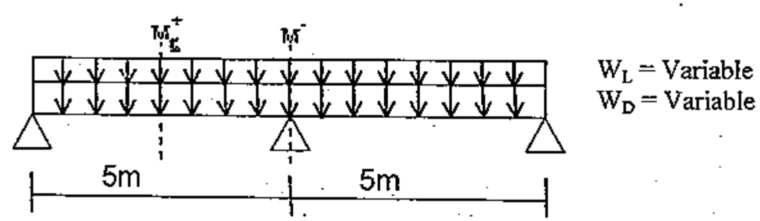


Table A.2-b Design of T cross-section and reinforcement of study number one (case 5)

2-span continuous beam (h_f=80mm, L=4m)

b(mm)	b_w(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	M^- ρ	M^- A_s(mm^2)	M^- ΦM_n(kN.m)	M^- M_u(kN.m)	M^+ ρ	M^+ A_s(mm^2)	M^+ ΦM_n(kN.m)	M^+ M_u(kN.m)
350	300	455	530	0.02125	0.25	0.00531	725	122.9	115.0	0.00416	530	88.5	57.5
		330	405		0.5	0.01063	1052	127.3	112.4		480	57.7	56.2
		275	350		0.75	0.01594	1315	130.9	111.4		580	57.1	55.7
350	300	540	615	0.02125	0.25	0.00531	861	173	161.6	0.00418	624	123.6	80.8
		390	465		0.5	0.01063	1243	178.4	158.6		570	80.9	79.3
		325	400		0.75	0.01594	1554	182.8	157.4		700	81.5	78.7
350	300	610	685	0.02125	0.25	0.00531	972	221.2	208.0	0.00951	2031	157.7	104.0
		440	515		0.5	0.01063	1403	226.9	204.6		1465	104.2	102.3
		370	445		0.75	0.01594	1769	237.1	203.2		1232	105.9	101.6



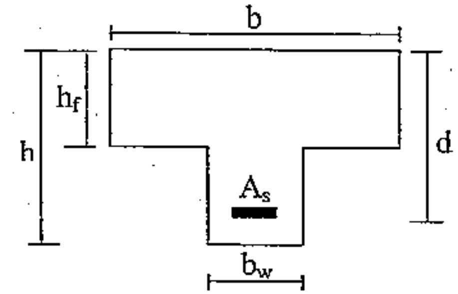
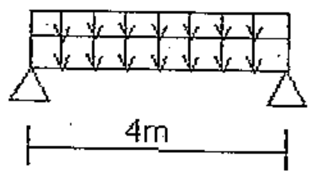


Table A.2-c Design of T cross-section and reinforcement of study number one (case 6)

2-span continuous beam (h_f=80mm, L=5m)

b(mm)	b_w(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	M^- ρ	M^- A_s(mm²)	M^- ΦM_n(kN.m)	M^- M_u(kN.m)	M^+ ρ	M^+ A_s(mm²)	M^+ ΦM_n(kN.m)	M^+ M_u(kN.m)
400	350	540	615	0.02125	0.25	0.00531	1004	201.9	192.6	0.00417	713	141.3	96.3
		390	465		0.5	0.01063	1450	203.1	187.0		650	91.2	93.5
		325	400		0.75	0.01594	1813	207.4	184.6		780	89.3	92.3
400	350	635	710	0.02125	0.25	0.00531	1181	279.5	259.4	0.01041	2643	195.3	129.7
		455	530		0.5	0.01063	1692	283.3	252.6		1894	132.3	126.3
		380	455		0.75	0.01594	2120	292.4	250.0		1582	129.2	125.0
450	400	670	745	0.02125	0.25	0.00531	1424	354.6	294.8	0.00925	2789	244.6	147.4
		480	555		0.5	0.01063	2040	359.1	326.8		1998	167.4	163.4
		400	475		0.75	0.01594	2550	369.4	339.4		1665	170.9	169.7



W_L = Variable
W_D = Variable

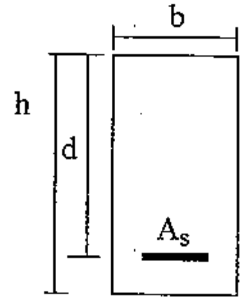
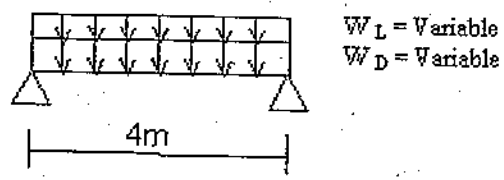


Table A.3-a Design of rectangular cross-section and reinforcement of study number two (case 1)
Simply supported beam (L=4m)

b(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)
300	355	430	0.02529	0.5	0.01264	1347	115.2	112.7
	305	380	0.03372		0.01686	1543	113.5	111.7
	280	355	0.04016		0.02008	1687	114.6	111.2
	375	450	0.01594		0.00797	897	115.1	113.1
	325	400	0.02125		0.01063	1036	115.5	112.1
	295	370	0.02531		0.01266	1120	113.8	111.5
300	420	495	0.02529	0.5	0.01264	1593	161.5	159.0
	365	440	0.03372		0.01686	1846	162.5	157.9
	330	405	0.04016		0.02008	1988	159.2	157.2
	445	520	0.01594		0.00797	1064	161.9	159.5
	385	460	0.02125		0.01063	1227	162.0	158.3
	350	425	0.02531		0.01266	1329	160.1	157.6



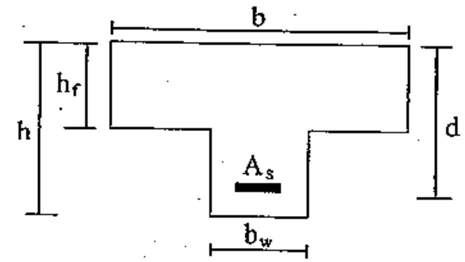


Table A.4-a Design of T cross-section and reinforcement of study number two (case 3)

Simply supported beam (h_f=80mm, L=4m)

b(mm)	b_w(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	$A_s(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)
1000	200	195	270	0.02529	0.5	0.01264	2466	116.0	111.9
		170	245	0.03372		0.01686	2866	117.6	111.6
		155	230	0.04016		0.02008	3113	117.0	111.4
		205	280	0.01594		0.00797	1634	114.8	112.1
		180	255	0.02125		0.01063	1913	117.9	111.7
		165	240	0.02531		0.01266	2088	118.7	111.5

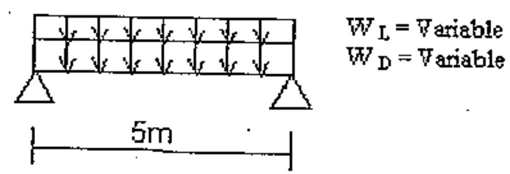
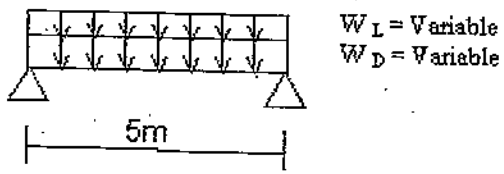


Table A.4-b Design of T cross-section and reinforcement of study number two (case 4)

Simply supported beam (h_f=80mm,L=5m)

b(mm)	b_w(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	$A_s(mm^2)$	ΦM_n(kN.m	M_u(kN.m)
1250	200	220	295	0.02529	0.5	0.01264	3477	184.6	177.5
		190	265	0.03372		0.01686	4004	184.6	176.9
		175	250	0.04016		0.02008	4393	185.8	176.6
		230	305	0.01594		0.00797	2291	180.5	177.7
		200	275	0.02125		0.01063	2656	181.5	177.1
		185	260	0.02531		0.01266	2927	185.6	176.8



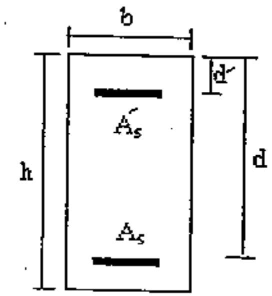
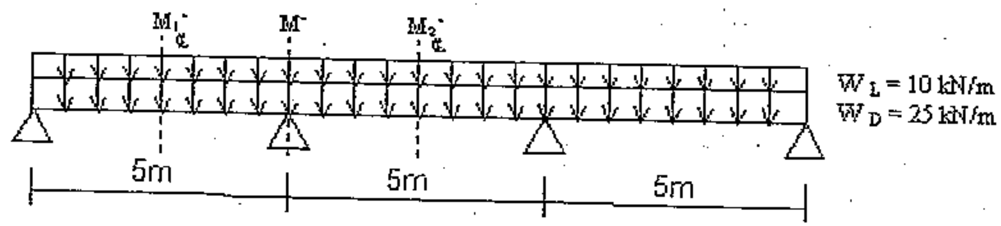


Table A.5-a Design of rectangular cross-section and reinforcement of study number three (case 1)
Simply supported beam (L=5m)

b(mm)	d(mm)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	$A_s(mm^2)$	$A_s'(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)
300	405	0.02125	0.5	0.01063	0.25	0.00266	1291	323	180.5	177.6
					0.5	0.00531		645	180.1	
					0.75	0.00797		968	181.1	
	335		0.75	0.01594	0.25	0.00398	1602	400	177.7	175.4
					0.5	0.00797		801	179.3	
					0.75	0.01195		1201	179.9	
300	480	0.02125	0.5	0.01063	0.25	0.00266	1530	383	254.2	250.3
					0.5	0.00531		765	255.5	
					0.75	0.00797		1148	256.2	
	395		0.75	0.01594	0.25	0.00398	1889	472	248.9	247.6
					0.5	0.00797		944	251.9	
					0.75	0.01195		1416	253.1	



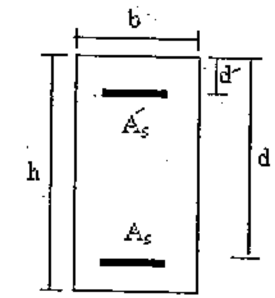
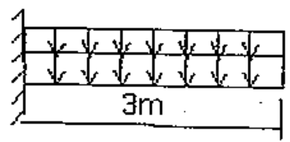


Table A.5-b Design of rectangular cross-section and reinforcement of study number three (case 2)
3-span continuous beam (L=5m)

				M^-								M + (Exterior Span)						M + (Interior span)			
b(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	$A_s(mm^2)$	$A_s'(mm^2)$	ϕM_n(kN.m)	M_u(kN.m)	ρ	ρ'	$A_s(mm^2)$	$A_s'(mm^2)$	ΦM_n(kN.m	M_u(kN.m)	ρ	$A_s(mm^2)$	ΦM_n(kN.m	M_u(kN.m)
300	360	435	0.02	0.5	0.011	0.25	0.003	1148	287	142.1	141.0	0.008	0.004	870	400	108.6	105.7	0.003	270	36.0	35.2
						0.5	0.005		574	142.2											
						0.75	0.008		861	142.3											
	300	375		0.75	0.016	0.25	0.004	1434	359	141.9	139.5	0.012	0.006	1060	550	107.2	104.6	0.004	330	36.2	34.9
						0.5	0.008		717	142.7											
						0.75	0.012		1076	143.2											
300	430	505	0.02	0.5	0.011	0.25	0.003	1371	343	203.4	199.0	0.008	0.004	1020	550	153.2	149.2	0.003	430	68.0	49.7
						0.5	0.005		685	204.4											
						0.75	0.008		1028	204.5											
	355	430		0.75	0.016	0.25	0.004	1697	424	199.9	197.1	0.012	0.007	1250	600	152.1	147.8	0.004	390	50.7	49.3
						0.5	0.008		849	201.8											
						0.75	0.012		1273	202.7											



$W_L = 10$ kN/m
$W_D = 25$ kN/m

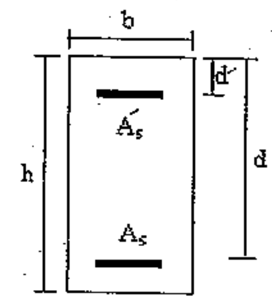
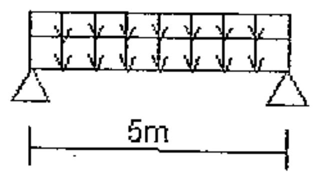


Table A.5-c Design of rectangular cross-section and reinforcement of study number three (case 3)
Cantilever beam (L=3m)

b(mm)	d(mm)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	$A_s(mm^2)$	$A_s'(mm^2)$	ΦM_n(kN.m)	M_u(kN.m)
300	490	0.02125	0.5	0.01063	0.25	0.00266	1562	390	265.0	259.6
					0.5	0.00531		781	266.3	
					0.75	0.00797		1171	267.0	
	405		0.75	0.01594	0.25	0.00398	1936	484	261.8	255.8
					0.5	0.00797		968	265.2	
					0.75	0.01195		1452	266.5	
300	575	0.02125	0.5	0.01063	0.25	0.00266	1833	458	366.7	364.7
					0.5	0.00531		916	295.8	
					0.75	0.00797		1375	300.9	
	475		0.75	0.01594	0.25	0.00398	2271	568	362.5	360.2
					0.5	0.00797		1136	368.0	
					0.75	0.01195		1703	370.6	



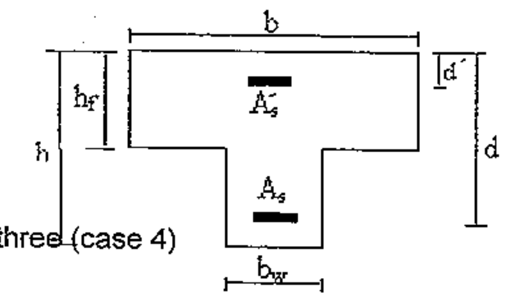


Table A.6-a Design of T cross-section and reinforcement of study number three (case 4)

Simply supported beam with compression steel (h_f=80mm, L=5m)

b(mm)	b_w(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	A_s(mm^2)	A_s'(mm^2)	ΦM_n(kN.m)	M_u(kN.m)
550	300	200	275	0	0.5	0.011	0.25	0.003	2656	664	182.7	177.1
							0.5	0.005		1328	182.6	
							0.75	0.008		1992	183.1	
		165	240		0.75	0.016	0.25	0.004	3287	822	176.3	176.4
							0.5	0.008		1644	176.3	
							0.75	0.012		2465	176.1	

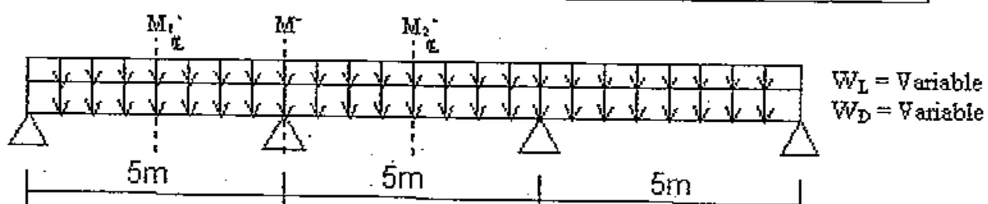


Table A.6-b Design of T cross-section and rei t of study number three (case 5)

3-span continuous beam with compression steel (h_f=80mm, L=5m)

b(mm)	b_w(mm)	d(mm)	h(mm)	ρ_{max}	ρ/ρ_{max}	ρ	ρ'/ρ	ρ'	A_s(mm^2)	A_s'(mm^2)	ϕM_n(kN.m)	M_u(kN.m)	ρ	A_s(mm^2)	ϕM_n(kN.m)	M_u(kN.m)	ρ	A_s(mm^2)	ΦM_n(kN.m)	M_u(kN.m)
550	250	400	475	0	0.5	0.011	0.3	0.003	1063	266	146.5	142.1	0.003	740	111.9	106.6	0.003	740	111.9	35.5
							0.5	0.005		531	147.0									
							0.8	0.008		797	147.2									
		330	405		0.8	0.016	0.3	0.004	1315	329	143.7	141.4	0.005	910	111.5	106.1	0.003	605	101.6	35.4
							0.5	0.008		657	144.8									
							0.8	0.012		986	145.3									

Appendix B

Computer Program Properties

This tailored program which is used to carry out the immediate deflection calculation of reinforced concrete beams has several properties.

- The software was developed using visual basic program. Previously developed models and procedures were modified and used to integrate the software. Only the procedures required to conduct the immediate deflection calculations were developed.
- The software calculates the deflection, using virtual method, at the mid span of (simple beams, continuous beams) and at fixed end of cantilever beams.
- Elastic analysis using stiffness approach was used.
- The program uses numerical integration, based on trapezoidal method, to conduct the virtual work procedure in calculating deflection.
- It considers simply supported beams, continuous beams with or without overhangs and cantilever beams.
- It considers T and rectangular sections in both analysis and deflection calculation.
- It considers uniformly distributed load over the span length.
- It considers tension and compression steel in the calculation of deflection.
- It can divide the element into a huge number of cross sections, according to the user needs.

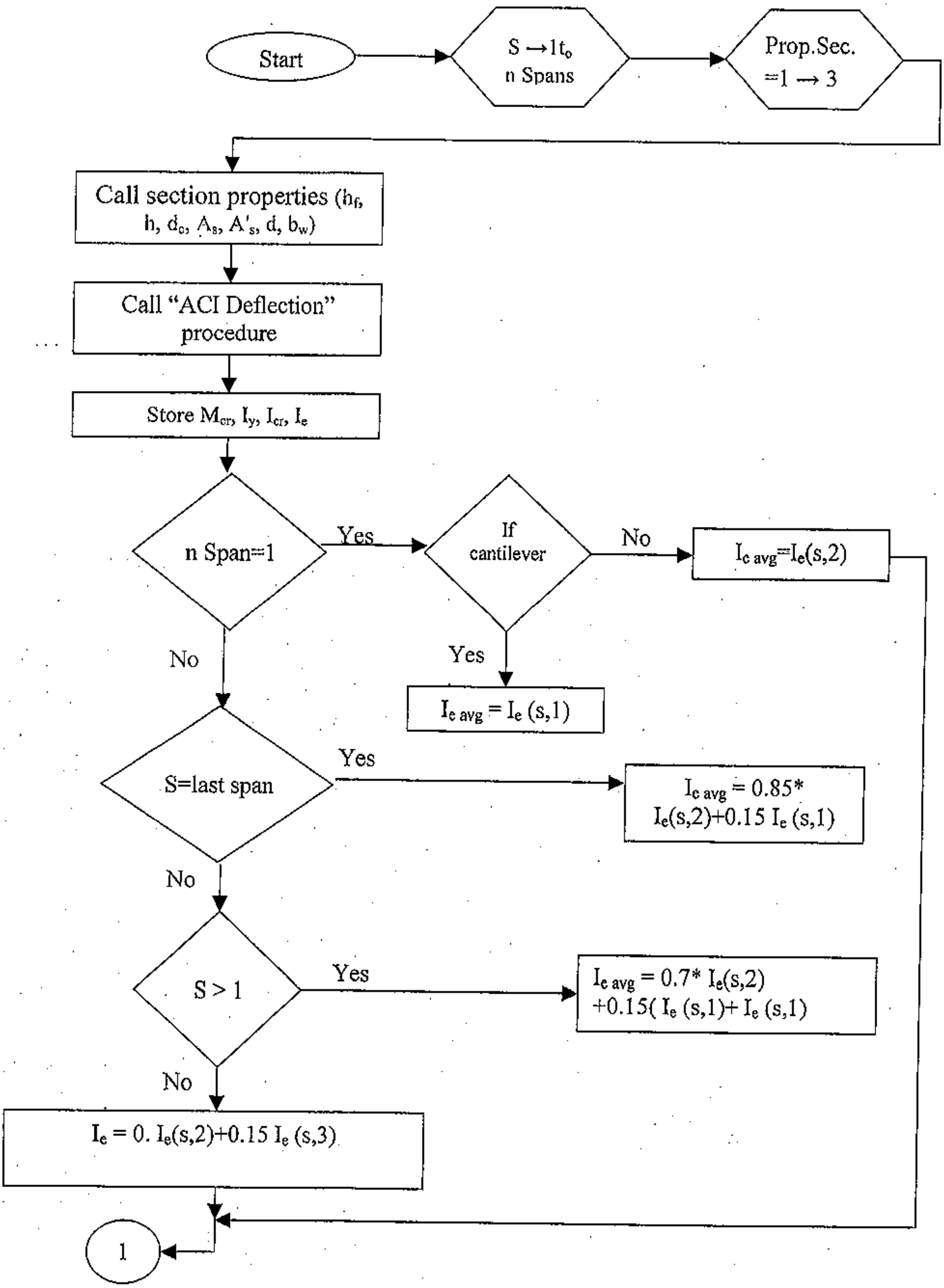


Figure B.1 Flow chart of computer program

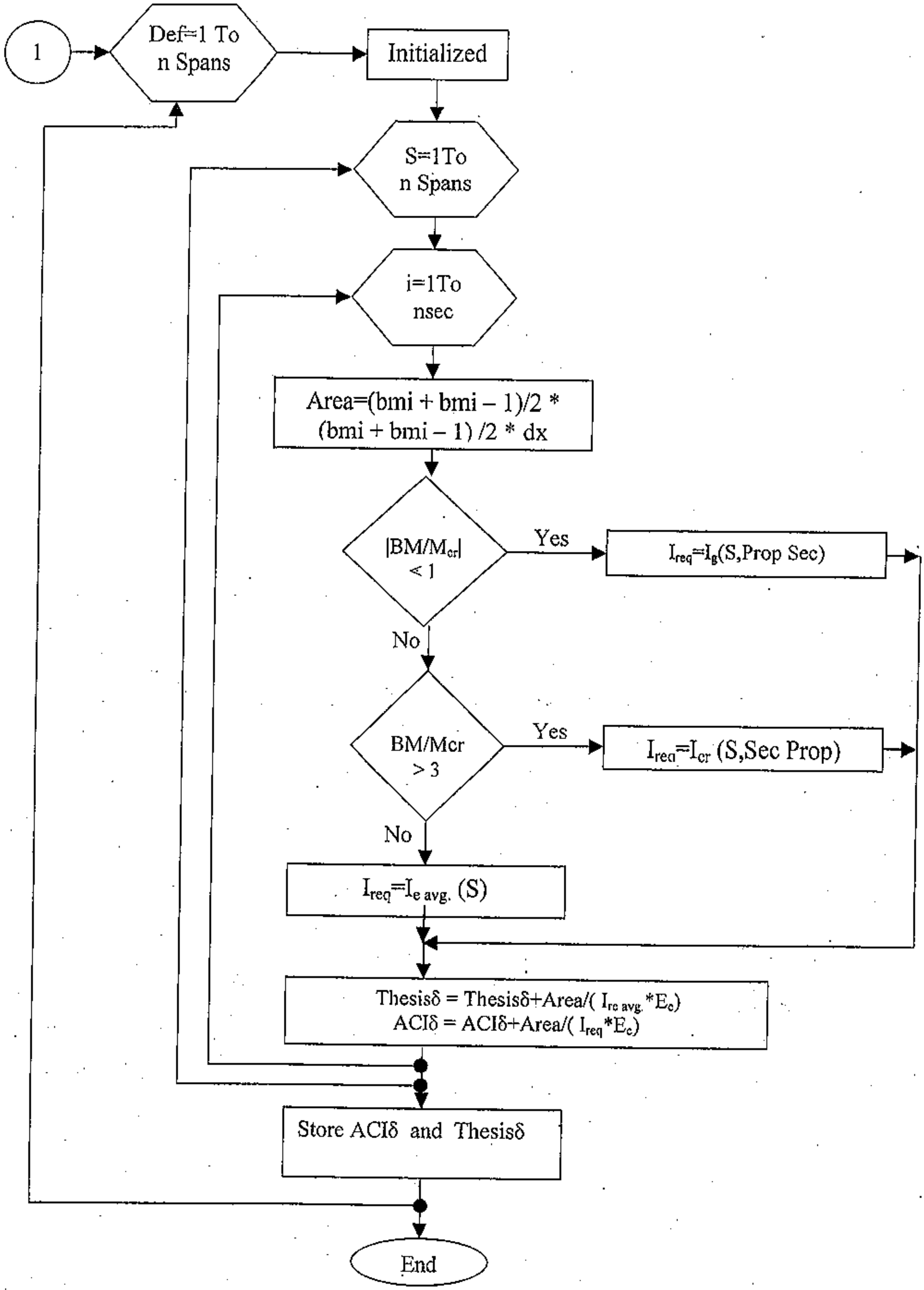


Figure B.1 Flow chart of computer program (continued)

الترخيم في الجسور الخرسانية المسلحة باستخدام عزم قصور ذاتي متغير

إعداد
وفاء عبد المجيد محمد

إشراف
أ. د. رائد السمرة

ملخص

تعــرض هــذه الرســالة طريقة عامة لتقدير الترخيم في منتصف الجسور الخرسانية المســلحة، والمعرضة لأحمال رأسية منتظمة التوزيع، آخذة بالإعتبار العوامل المختلفة التي تؤثر في ترخيم الجسور.

تشــمل الدراســة تأثيــر كل من: طول بحر الجسر، نسبة حديد التسليح في كل من منطقــتي الشد والضغط، مقدارالحمل المؤثر على الجسر، قوة إجهاد خضوع الفولاذ، مقاومة الخرسانة للضغط، نوع الجسر والمقطع العرضي للجسر.

تم عمل مقارنة بين الترخيم المحسوب بواسطة الطريقة المحسوبة في الرسالة مع تلك التي يحددها الكود الأمريكي. وجد تقارب كبير في كل الحالات ما عدا الجسور المستمرة التي تحتوي على نسب حديد تسليح مرتفعة في منطقة الشد.

تــتأثر العلاقة بين نتائج الطريقة المقترحة والكود الأمريكي بمتغيرات عديدة، توضح الدراسة أن نتائج الحسابات في الطريقتين تكون أكثر تقارباً في حالات الجسر البسيط الإرتكاز والجسر الكابولي والجسر المستمر ذو البحرين و الجسر المستمر ذو الثلاثة أبحر الذي يحتوي على نسبة حديد تسليح ($\rho < 0.75\ \rho_{max}$).